

## MAT2540 Project Spring2026

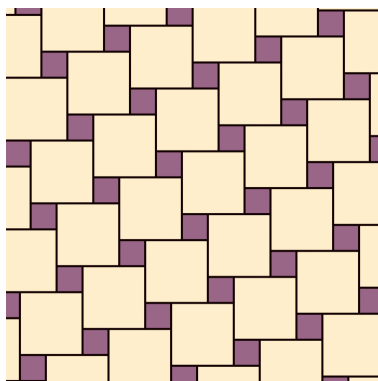
1. In section 5.2, we introduce the Mathematical induction which could be used to prove statements “ $P(n)$  is true” for all positive integers where  $P(n)$  is a propositional function.

Use the induction to prove the following statement:

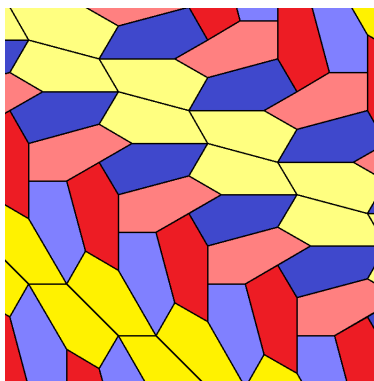
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ for all } n \in \mathbb{Z}^+$$

2. Introduction of Tessellation.

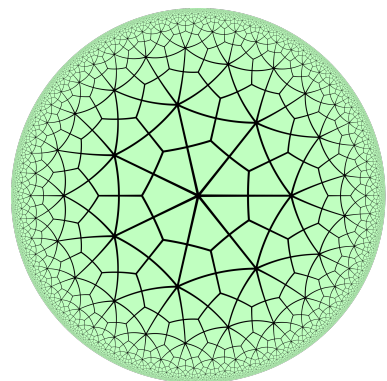
A tessellation or tiling is the covering of a surface using one or more geometric shapes with no overlaps and no gaps.



(a) A Pythagorean tiling



(b) A convex monohedral pentagonal tiling



(c) A deltoidal triheptagonal tiling

Figure 1: Examples of tessellation

From 1., we know

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ for all } n \in \mathbb{Z}^+.$$

If we rewrite this statement as the following:

$$1 \times 1^2 + 2 \times 2^2 + 3 \times 3^2 + \cdots + n \times n^2 = 1 \times \left(\frac{n(n+1)}{2}\right)^2,$$

and see the square numbers as the areas of squares ( $1^2, 2^2, 3^2, \dots, n^2$ , and  $\left(\frac{n(n+1)}{2}\right)^2$  are the area of squares with length  $1, 2, 3, \dots, n$ , and  $\frac{n(n+1)}{2}$ , respectively).

Theoretically speaking, for a given integer number  $n$ , there is a tessellation of a  $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$  square by using an  $1 \times 1$  square, two of  $2 \times 2$  squares, three of  $3 \times 3$  squares,  $\dots$ , and  $n$  of  $n \times n$  squares to cover it.

For example, a possible tessellation for  $n = 8$  is in Figure 2. This is a  $36 \times 36$  square covered by an  $1 \times 1$  square, two of  $2 \times 2$  squares, three of  $3 \times 3$  squares, four of  $4 \times 4$  squares, five of  $5 \times 5$  squares, six of  $6 \times 6$  squares, seven of  $7 \times 7$  squares, and eight of  $8 \times 8$  squares.

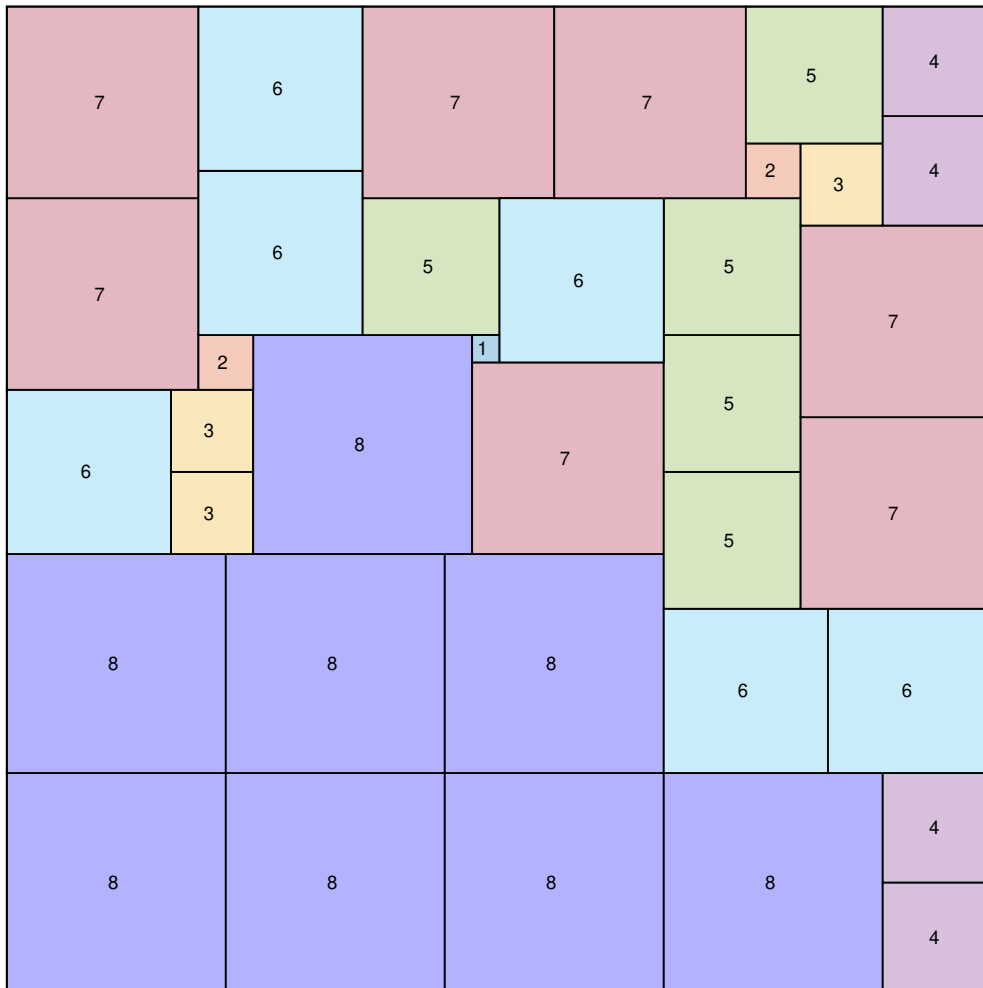


Figure 2: A square tessellation of a  $36 \times 36$  square.

Please provide a pseudocode of the algorithm to find this kind of tessellation.

- Write a code based on your algorithm to find this kind of tessellations. Please provide a tessellation for  $n = 9$ .