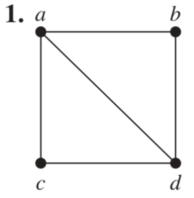
Mat2540 HW12

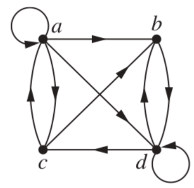
In Exercises 1–4 use an adjacency list to represent the given graph.

1.



Vertex	Adjacent Vertices
a	b, c, d
b	a, d
c	a, d
d	a, b, c

3.



Vertex	Adjacent Vertices
a	a, b, c, d
b	d
c	a, b
d	b, c, d

5. Represent the graph in Exercise 1 with an adjacency matrix.

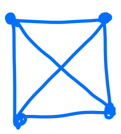
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

7. Represent the graph in Exercise 3 with an adjacency matrix.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

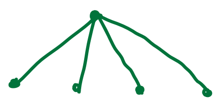
9. Represent each of these graphs with an adjacency matrix.

a) K_4



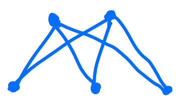
$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

b) $K_{1,4}$



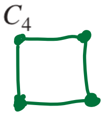
$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c) $K_{2,3}$



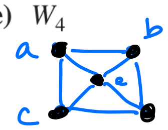
$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

d) C_4



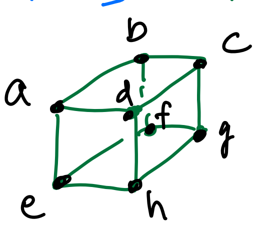
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

e) W_4



$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

f) Q_3

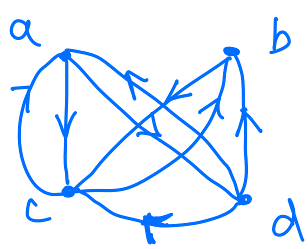


$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

11.

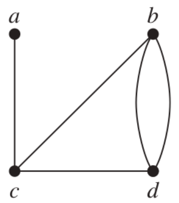
$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

NOT symmetric ⇒ directed graph, looking for out-degree row-wise.



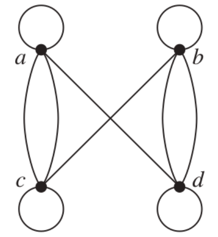
In Exercises 13–15 represent the given graph using an adjacency matrix.

13.



$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

15.

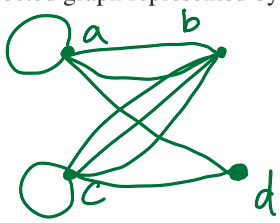


$$\begin{bmatrix} 2 & 0 & 2 & 1 \\ 0 & 2 & 1 & 2 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 0 & 2 \end{bmatrix}$$

In Exercises 16–18 draw an undirected graph represented by the given adjacency matrix.

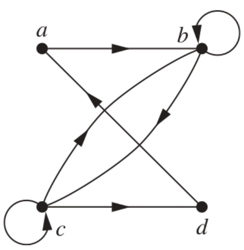
17.

$$\begin{array}{cccc} a & b & c & d \end{array}$$
$$\begin{bmatrix} 2 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 2 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$



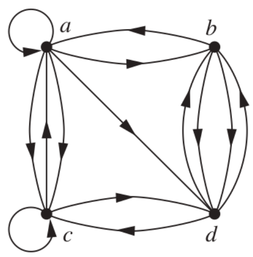
In Exercises 19–21 find the adjacency matrix of the given directed multigraph with respect to the vertices listed in alphabetic order.

19.



$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

21.



$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 0 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

In Exercises 22–24 draw the graph represented by the given adjacency matrix.

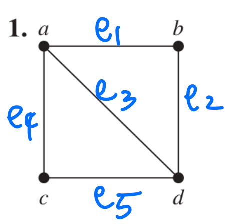
23.

$$\begin{array}{ccc} a & b & c \end{array}$$
$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

NOT symmetric ⇒ Directed graph

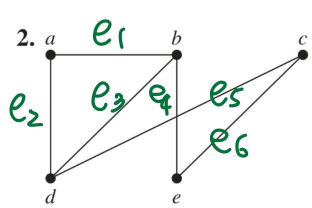
30. Use an incidence matrix to represent the graphs in Exercises 1 and 2.

1.



$$\begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline a & 1 & 0 & 1 & 1 & 0 \\ b & 1 & 1 & 0 & 0 & 0 \\ c & 0 & 0 & 0 & 1 & 1 \\ d & 0 & 1 & 1 & 0 & 1 \end{array}$$

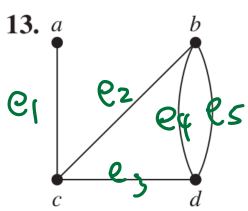
2.



$$\begin{array}{c|cccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ \hline a & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 0 & 1 & 1 & 0 & 0 \\ c & 0 & 0 & 0 & 1 & 1 & 1 \\ d & 0 & 1 & 1 & 0 & 1 & 0 \\ e & 0 & 0 & 0 & 1 & 0 & 1 \end{array}$$

31. Use an incidence matrix to represent the graphs in Exercises 13–15.

13.



$$\begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline a & 1 & 0 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 1 & 1 \\ c & 1 & 1 & 1 & 0 & 0 \\ d & 0 & 0 & 1 & 1 & 1 \end{array}$$

34. What is the sum of the entries in a row of the incidence matrix for an undirected graph?

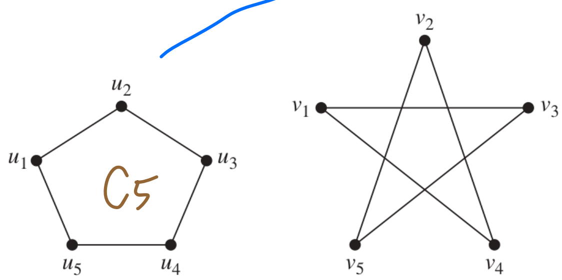
The degree of that vertex.

35. What is the sum of the entries in a column of the incidence matrix for an undirected graph?

The two endpoints for the certain edge.

In Exercises 38–48 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. For additional exercises of this kind, see Exercises 3–5 in the Supplementary Exercises.

39.



$|V|=5$, $|E|=5$, degree sequence: 2, 2, 2, 2, 2

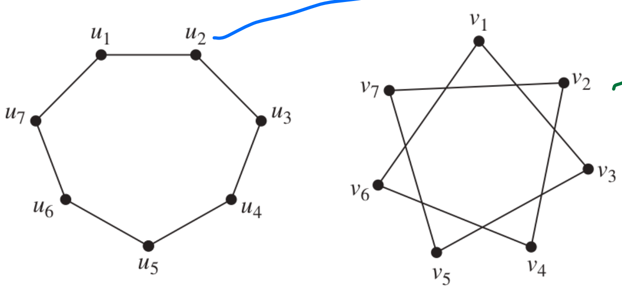
$|V|=5$, $|E|=5$, degree sequence 2, 2, 2, 2, 2

The isomorphism exists:

$f(u_1)=v_1$, $f(u_2)=v_3$, $f(u_3)=v_5$

$f(u_4)=v_2$, $f(u_5)=v_4$

41.



$|V|=7$, $|E|=7$, degree sequence 2, 2, 2, 2, 2, 2, 2

$|V|=7$, $|E|=7$, degree sequence 2, 2, 2, 2, 2, 2, 2

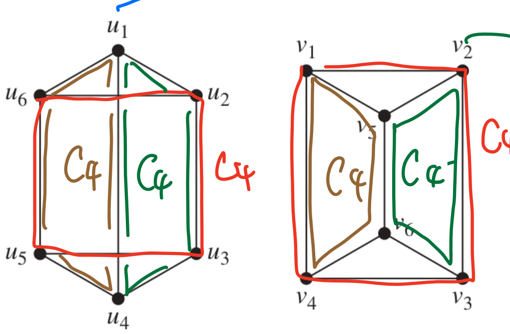
The isomorphism exists

$f(u_1)=v_1$, $f(u_2)=v_3$, $f(u_3)=v_5$

$f(u_4)=v_7$, $f(u_5)=v_2$, $f(u_6)=v_4$

$f(u_7)=v_6$

43.



$|V|=6$, $|E|=9$, degree sequence 3, 3, 3, 3, 3, 3

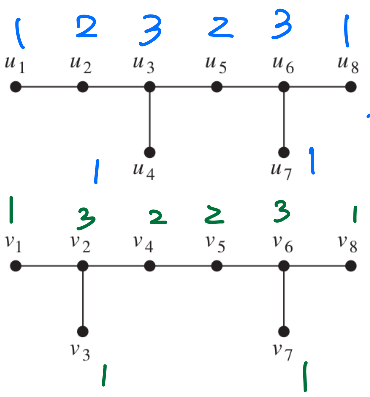
$|V|=6$, $|E|=9$, degree sequence 3, 3, 3, 3, 3, 3

The isomorphism exists

$f(u_1)=v_5$, $f(u_2)=v_2$, $f(u_3)=v_3$

$f(u_4)=v_6$, $f(u_5)=v_4$, $f(u_6)=v_1$

45.



$|V|=8$, $|E|=7$, degree sequence 3, 3, 2, 2, 1, 1, 1, 1

$|V|=8$, $|E|=7$, degree sequence 3, 3, 2, 2, 1, 1, 1, 1

The isomorphism does not exist: if it existed, $f(u_3)=v_2$, but the neighbors of u_3 have 2, 2 as degrees, and v_2's neighbors have

1, 2 as their degrees.