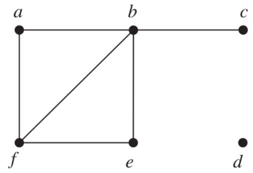
Mat2540 HW11

Let $|V| = \#$ of vertices
$|E| = \#$ of edges

4. Find the sum of the degrees of the vertices of each graph in Exercises 1–3 and verify that it equals twice the number of edges in the graph.

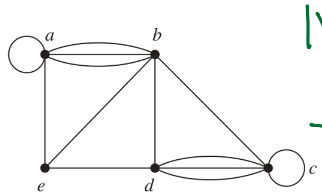
In Exercises 1–3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

1. 

$|V| = 6$ $|E| = 6$, d is isolated
c is pendant

	a	b	c	d	e	f
deg	2	3	1	0	2	3

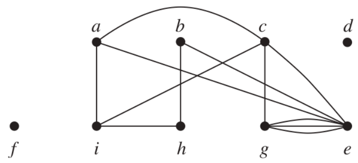
$\Rightarrow$ sum of degree $= 12$ and it equals $2|E|$

2. 

$|V| = 5$, $|E| = 13$ NO isolated or pendant vertex

	a	b	c	d	e
deg	6	6	6	5	3

$\Rightarrow$ sum of degree $= 26$ and it equals $2|E|$

3. 

$|V| = 9$, $|E| = 12$, f, d is isolated

	a	b	c	d	e	f	g	h	i
deg	3	2	4	0	6	0	4	2	3

$\Rightarrow$ sum of degree $= 24$ and it equals $2|E|$

5. Can a simple graph exist with 15 vertices each of degree five?

If there were a graph with 15 vertices each of degree 5, then the sum of degree is $15 \cdot 5 = 75$.
Since the half of sum of degree equals the number of edges, it implies there were $\frac{75}{2} = 37.5$ edges which is impossible.
Therefore, this graph doesn't exist.

6. Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

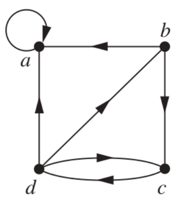
We can build an undirected graph and put an edge between two people if they shake hand with each other and "the degree of a person" represents the number of people this certain person has shaken hands with.
Since the sum of the degrees is an even, then the sum of shanking hands is even as well.

In Exercises 7–9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

10. For each of the graphs in Exercises 7–9 determine the sum of the in-degrees of the vertices and the sum of the out-degrees of the vertices directly. Show that they are both equal to the number of edges in the graph.

Let $|V|$ = # of vertices, $|E|$ = # of edges

7.

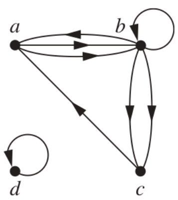


$|V|=4$, $|E|=7$

	a	b	c	d
in-deg	3	1	2	1
out-deg	1	2	1	3

sum of in-degree = 7
$\Rightarrow$ sum of out-degree = 7
both equal $|E|$

8.

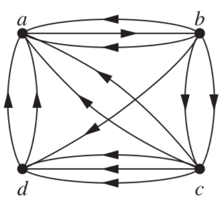


$|V|=4$, $|E|=8$

	a	b	c	d
in-deg	2	3	2	1
out-deg	2	4	1	1

sum of in-degree = 8
$\Rightarrow$ sum of out-degree = 8
and both equal $|E|$

9.



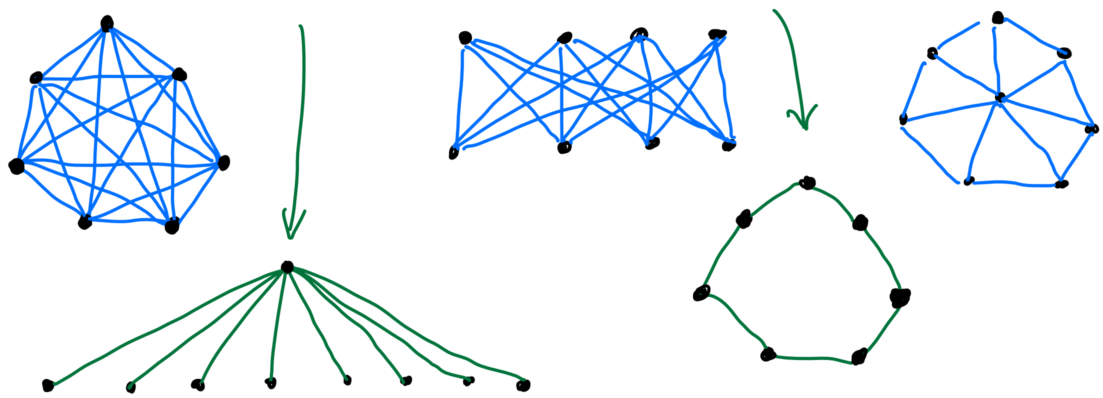
$|V|=4$, $|E|=13$

	a	b	c	d
in-deg	6	1	2	4
out-deg	1	5	5	2

sum of in-degree = 13
$\Rightarrow$ sum of out-degree = 13
and both equal $|E|=13$

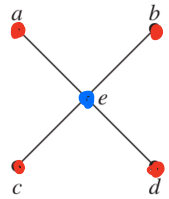
20. Draw these graphs.

a) K_7 b) $K_{1,8}$ c) $K_{4,4}$ d) C_7 e) W_7 f) Q_4



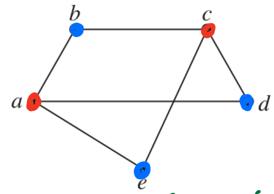
In Exercises 21–25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

21.



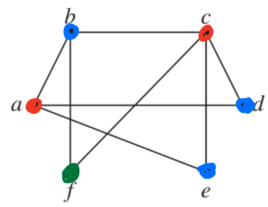
2-colorable
$\Rightarrow$ It's bipartite

22.



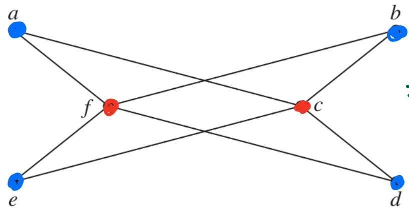
2-colorable
$\Rightarrow$ It's bipartite

23.



Not 2-colorable
$\Rightarrow$ It's not bipartite.

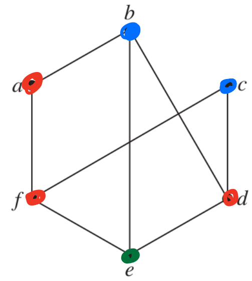
24.



2-colorable

$\Rightarrow$ It's bipartite

25.



NOT 2-colorable

$\Rightarrow$ It's not bipartite.

37. How many vertices and how many edges do these graphs have?

a) K_n **b)** C_n **c)** W_n **d)** $K_{m,n}$ **e)** Q_n

a) K_n has n vertices and $\frac{n(n-1)}{2}$ edges

b) C_n has n vertices and n edges

c) W_n has $n+1$ vertices and $2n$ edges

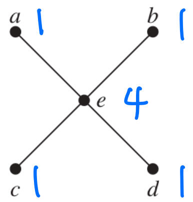
d) $K_{m,n}$ has $m+n$ vertices and $m \cdot n$ edges

e) Q_n has 2^n vertices and $\frac{n \cdot 2^n}{2}$ edges $\rightarrow n \cdot 2^{n-1}$

The **degree sequence** of a graph is the sequence of the degrees of the vertices of the graph in nonincreasing order. For example, the degree sequence of the graph G in Example 1 is 4, 4, 4, 3, 2, 1, 0.

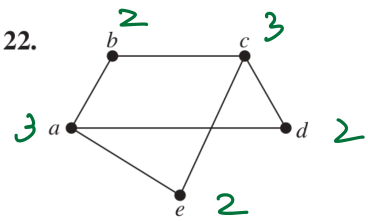
38. Find the degree sequences for each of the graphs in Exercises 21–25.

21.



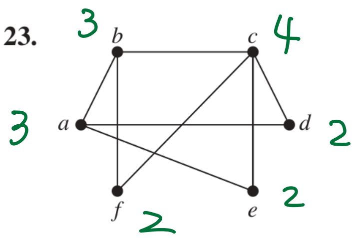
$\Rightarrow$ 4, 1, 1, 1, 1

22.



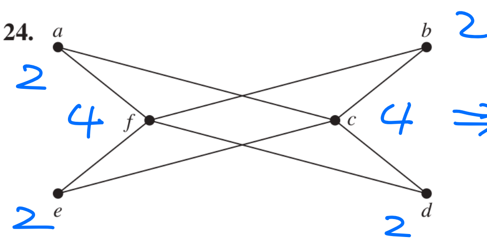
$\Rightarrow$ 3, 3, 2, 2, 2

23.



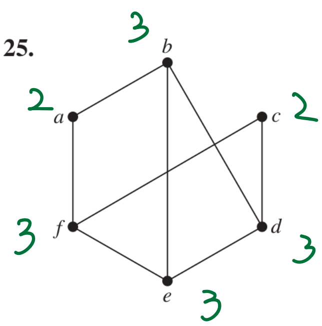
$\Rightarrow$ 4, 3, 3, 2, 2, 2

24.



$\Rightarrow$ 4, 4, 2, 2, 2, 2

25.



$\Rightarrow$ 3, 3, 3, 3, 2, 2

A sequence $d_1, d_2, \ldots, d_n$ is called **graphic** if it is the degree sequence of a simple graph.

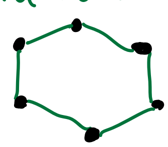
44. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.

a) 5, 4, 3, 2, 1, 0 **b)** 6, 5, 4, 3, 2, 1 **c)** 2, 2, 2, 2, 2, 2

a) NOT Graphic. There are six vertices and it is impossible for a vertex with 5 edges when there is an isolated vertex (0 degree) in a simple graph

b) NOT Graphic. There are six vertices and it is impossible for a vertex with degree 6 (which has 6 edges)

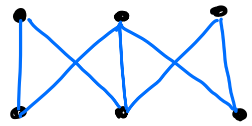
c) Graphic and it is a C_6



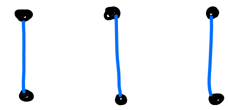
d) 3, 3, 3, 2, 2, 2 **e)** 3, 3, 2, 2, 2, 2 **f)** 1, 1, 1, 1, 1, 1

d) NOT Graphic, sum of degree is an odd number which is impossible

e) Graphic

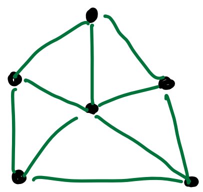


f) Graphic



g) 5, 3, 3, 3, 3, 3 **h)** 5, 5, 4, 3, 2, 1

g)



Graphic which is a W_5

h) Not Graphic.

There are six vertices and two vertices are have 5 edges but it is imposible to have one vertex with only one edge.

61. The **complementary graph** $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. Describe each of these graphs.

a) $\overline{K_n}$ **b)** $\overline{K_{m,n}}$ **c)** $\overline{C_n}$ **d)** $\overline{Q_n}$

a) K_n is a complete graph and there is no way to add more edge(s), Thus, $\overline{K_n} = \phi$

b) $K_{m,n}$ is bipartite with m vertices have edges with each vertex from all the other n vertices. Then $\overline{K_{m,n}}$ is a graph combining with a K_m and K_n.

c) $\overline{C_n} = K_n - C_n$