MAT2540, Quiz1, Spring2026

ID:_____ Name: Sol. 02/03/2026

- This quiz consists of 1 question for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.

1. Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps.

Use **strong induction** to prove that $P(n)$ is true for all integers $n \geq 18$.

Basis Step: To show $P(18), P(19), P(20), P(21)$ are true, we have

$18 = 7+7+4$, $19 = 7+4+4+4$, $20 = 4+4+4+4+4$, $21 = 7+7+7$.

which means these four cases are true

Inductive Step: Assume $P(j)$ is true for $18 \leq j \leq k$ with $k \geq 21$.

To prove $P(k+1)$ is true, we have

since $k \geq 21$, then $k-3 \geq 18$ and it means ($k-3$ is one of the j's)

$P(k-3)$ is true, that is,

$k-3$ can be formed by 4-cent and 7-cent stamps.

Therefore, $k+1 = k-3+4$ can be formed by $k-3$ with one extra 4-cent stamp.

It means $k+1$ can be formed by 4-cent and 7-cent stamps.

Hence, $P(k+1)$ is true.

Conclusion: $P(n)$ is true for all $n \geq 18$.