

# MAT2540, Classwork13, Spring2026

## 10.1 Graphs and Graph Models

### 1. Definition: Undirected Graphs.

A graph  $G = (V, E)$  consists of  $V$ , a nonempty set of vertices (or nodes) and  $E$ , a set of edges. Each edge has either one or two vertices associated with it, called its endpoints. An edge is said to connect its endpoints.

Infinite graph: a graph with an infinite vertex set or an infinite number of edges.

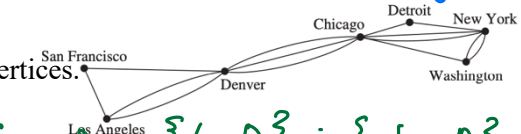
Finite graph: a graph with a finite vertex set and a finite edge set.



### 2. Simple graph and Multigraphs.

A graph is called a **simple graph** if each edge connects two different vertices and no two edges connect the same pair of vertices.

A graph is called a **multigraph** if it has multi edges connects two different vertices.

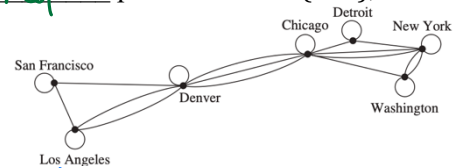


### 3. The representations of the edge.

When there is an edge of a **simple graph** associated to  $\{u, v\}$ , we can also say that  $\{u, v\}$  is an edge of the graph

In a **multigraph**, when there are  $m$  different edges associated to the same unordered pair of vertices  $\{u, v\}$ ,

we also say that  $\{u, v\}$  is an edge of multiplicity  $m$ .



### 4. The Pseudograph and the loops.

An edge is called a loop if it connects a vertex to itself, and it may have more than one loop at a vertex.

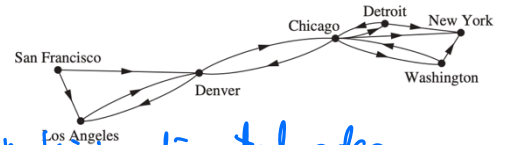
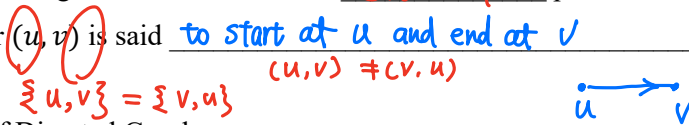
Graphs that include loops and multiple edges connecting the same pair of vertices or a vertex to itself are called pseudograph.

### 5. Definition: Directed Graphs.

A directed graph (or digraph)  $(V, E)$  consists of a nonempty set of vertices  $V$  and a set of directed edges (or arcs)  $E$ .

Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the

ordered pair  $(u, v)$  is said to start at  $u$  and end at  $v$ .

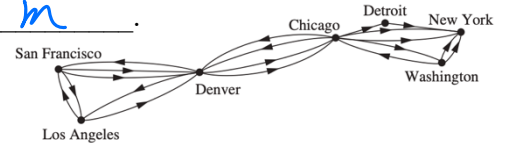


### 6. The types of Directed Graphs.

*Simple directed graph:* a directed graph has no loop and has no multiple directed edge.

*Directed multigraphs:* Directed graphs that may have multiple directed edges from a vertex to a second (possibly the same) vertex are used to model such networks. When there are  $m$  directed edges, each associated to an ordered pair of vertices  $(u, v)$ , we say that  $(u, v)$  is an edge with multiplicity  $m$ .

*Mixed graph:* A graph with both directed and undirected edges



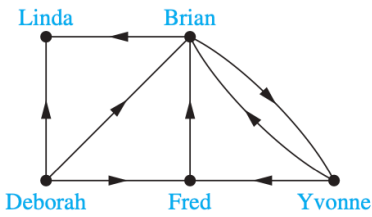
### 7. Graph Terminology

Type	Directed Edges?	Multiple Edges Allowed?	Loops Allowed?
Simple graph	undirected	NO	NO
Multigraph	undirected	Yes	NO
Pseudograph	undirected	Yes	Yes
Simple directed graph	directed	NO	NO
Directed multigraph	directed	Yes	Yes
Mixed graph	undirected and directed	Yes	Yes

8. Three key questions can help us understand the structure of a graph:

- 1) Are the edges of the graph undirected or directed (or both)?
- 2) If the graph is undirected, are multiple edges present that connect the same pair of vertices? If the graph is directed, are multiple edge present.
- 3) Are loop present?

9. Example of Social Networks: Influence Graphs.



Is this graph a directed or undirected graph? Directed

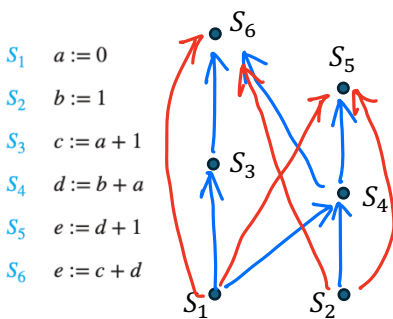
Each person of the group is represented by a vertex.

There is a directed edge from vertex *a* to vertex *b* when the person represented by Vertex a can influence the person represented by vertex b

In this graph, who cannot be influenced? Deborah

In this graph, who can influence each other? Brian and Yvonne

10. Example of Software Design Application: Precedence Graphs and Concurrent Processing



- $S_1$   $a := 0$
- $S_2$   $b := 1$
- $S_3$   $c := a + 1$
- $S_4$   $d := b + a$
- $S_5$   $e := d + 1$
- $S_6$   $e := c + d$

Is this graph a directed or undirected graph? Directed

It can be executed more rapidly by executing certain statements concurrently.

Each statement is represented by a vertex, and there is an edge from one statement to a second statement if the second statement cannot be executed before the first one

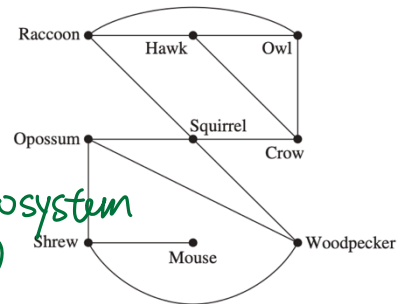
11. Example of Graph Model: Niche Overlap Graphs in Ecology.

Niche overlap graph: the competition between species in ecosystem

Is this graph a directed or undirected graph? undirected (since it is mutual)

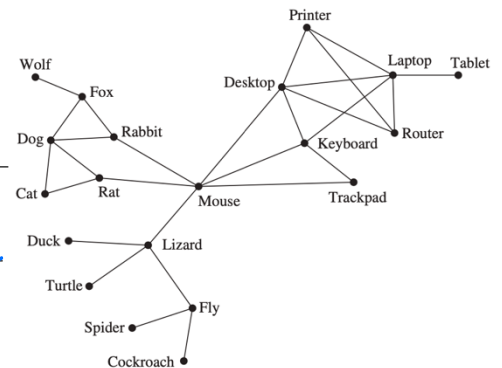
Is this graph a simple or multigraph? simple

Each species is represented by a vertex. An undirected edge connects two vertices if the two species represented by these vertices compete

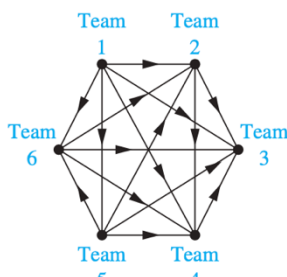


12. Example of Graph Model: Semantic Networks

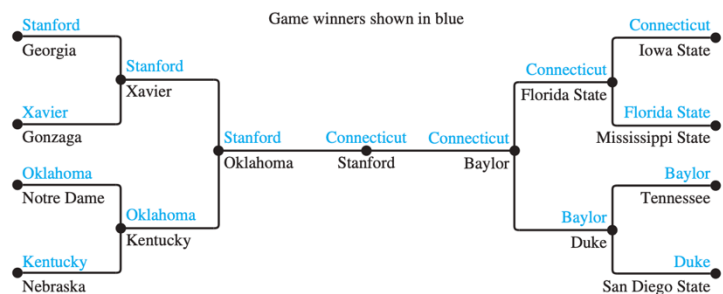
In semantic networks, vertices are used to represent words, and undirected edges are used to connect vertices when a semantic relation holds between these words. A semantic relation is a relationship between two or more words that is based on the meaning of the words



13. Example of Graph Model: Tournaments.



A round-robin tournament.



A single-elimination tournament