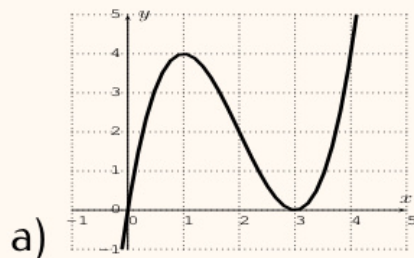
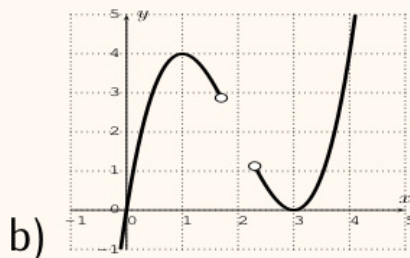
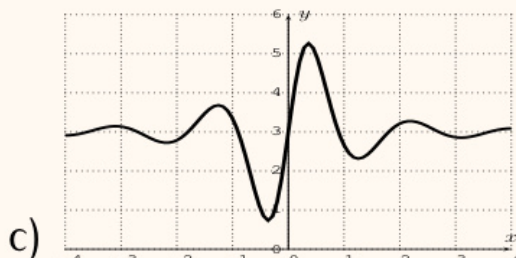
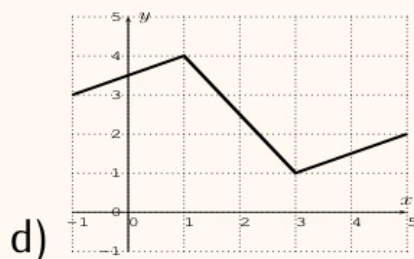
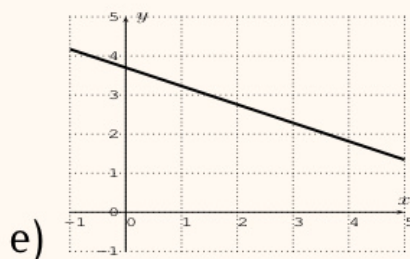


Mat 1375 HW8

Exercise 8.1

Assuming the graphs below are complete graphs, which of the graphs could be the graphs of a polynomial?



Sol (a) Polynomial

(b) Not a polynomial (discontinuous)

(c) Polynomial

(d) Not a Polynomial (Not smooth)

(e) Polynomial

(f) Polynomial

Exercise 8.2

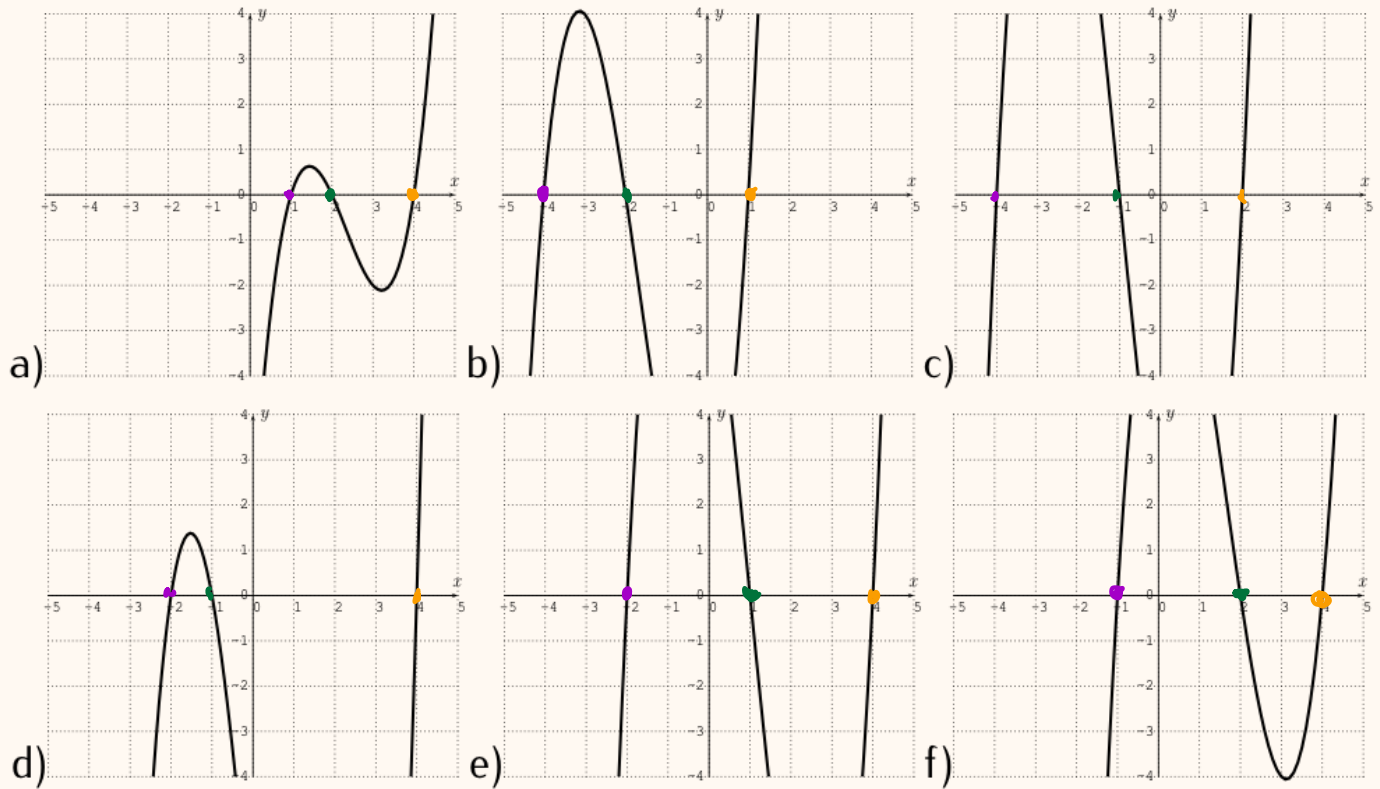
For each of the polynomials f , g , h , and k , find the corresponding graph from (a)-(f) below.

$$f(x) = (x - 1) \cdot (x + 2) \cdot (x - 4)$$

$$g(x) = (x + 1) \cdot (x - 2) \cdot (x + 4)$$

$$h(x) = (x - 1) \cdot (x - 2) \cdot (x - 4)$$

$$k(x) = (x + 1) \cdot (x - 2) \cdot (x - 4)$$



Sol

a) roots (x-intercepts): $x=1$, $x=2$, $x=4$

$$\text{Polynomial} = (x-1) \cdot (x-2) \cdot (x-4) \Rightarrow h(x)$$

b) roots: $x=-4$, $x=-2$, $x=1$

$$\text{Polynomial} = (x+4) \cdot (x+2) \cdot (x-1) \Rightarrow \text{none of above}$$

c) roots: $x=-4$, $x=-1$, $x=2$

$$\text{Polynomial} = (x+4) \cdot (x+1) \cdot (x-2) \Rightarrow g(x)$$

d) roots: $x=-2$, $x=-1$, $x=4$

$$\text{Polynomial} = (x+2) \cdot (x+1) \cdot (x-4) \Rightarrow \text{none of above}$$

e) roots: $x = -2$, $x = 1$, $x = 4$

$$\text{Polynomial} = (x+2) \cdot (x-1) \cdot (x-4) \Rightarrow f(x)$$

f) roots $x = -1$, $x = 2$, $x = 4$

$$\text{Polynomial} = (x+1) \cdot (x-2) \cdot (x-4) \Rightarrow k(x).$$

Exercise 8.3

For each of the polynomials f , g , h , and k , find the corresponding graph from (a)-(f) below.

$$f(x) = (x+1) \cdot (x+2)^2$$

$$g(x) = -(x+1) \cdot (x-2)^2$$

$$h(x) = -(x-1)^2 \cdot (x+2)$$

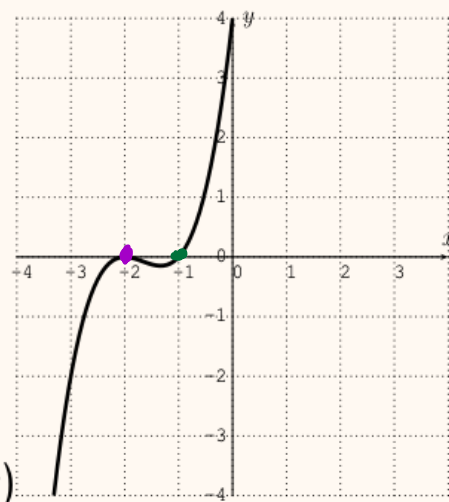
$$k(x) = (x-1) \cdot (x+2)^2$$



a)



b)



c)

Sol:

a) Roots: $x = -2$, $x = 1$, $x = 1$ (multiplicity = 2)

$$\text{Polynomial} = + (x+2) \cdot (x-1)^2 \Rightarrow \text{None of above}$$

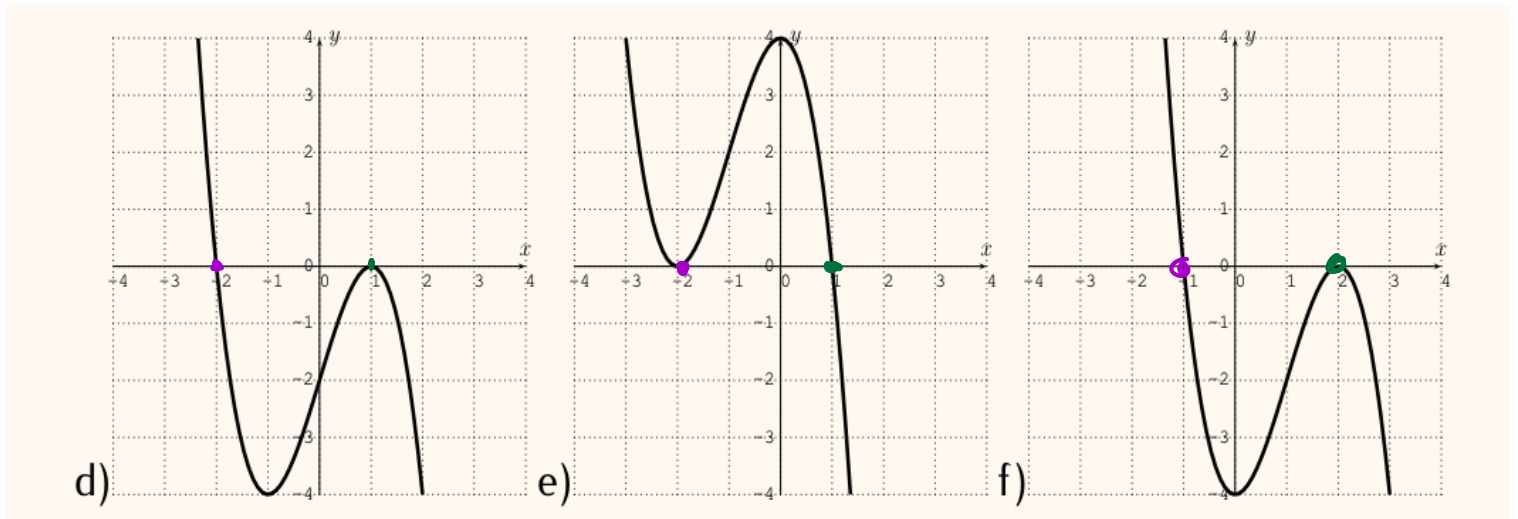
(end behavior $(\downarrow, \uparrow) \Rightarrow$ leading coeff > 0)

b) Roots: $x = -2$, $x = -2$ (multiplicity = 2), $x = 1$

$$\text{Polynomial} = + (x+2)^2 \cdot (x-1) \Rightarrow k(x)$$

(end behavior $(\downarrow, \uparrow) \Rightarrow$ leading coeff > 0)

c) Roots: $x = -2, x = -2$ (multiplicity = 2), $x = -1$
 Polynomial = $+(x+2)^2 \cdot (x+1) \Rightarrow f(x)$
 (end behavior $(\downarrow, \uparrow) \Rightarrow$ leading coeff > 0)



d) Roots: $x = -2, x = 1, x = 1$ (multiplicity = 2)
 Polynomial = $-(x+2) \cdot (x-1)^2 \Rightarrow h(x)$
 (end behavior $(\uparrow, \downarrow) \Rightarrow$ leading coeff < 0)

e) Roots: $x = -2, x = -2$ (multiplicity = 2), $x = 1$
 Polynomial = $-(x+2)^2 (x-1) \Rightarrow$ None of above.
 (end behavior $(\uparrow, \downarrow) \Rightarrow$ leading coeff < 0)

f) Roots: $x = -1, x = 2, x = 2$
 Polynomial = $-(x+1) \cdot (x-2)^2 \Rightarrow g(x)$
 (end behavior $(\uparrow, \downarrow) \Rightarrow$ leading coeff < 0)

Exercise 8.4

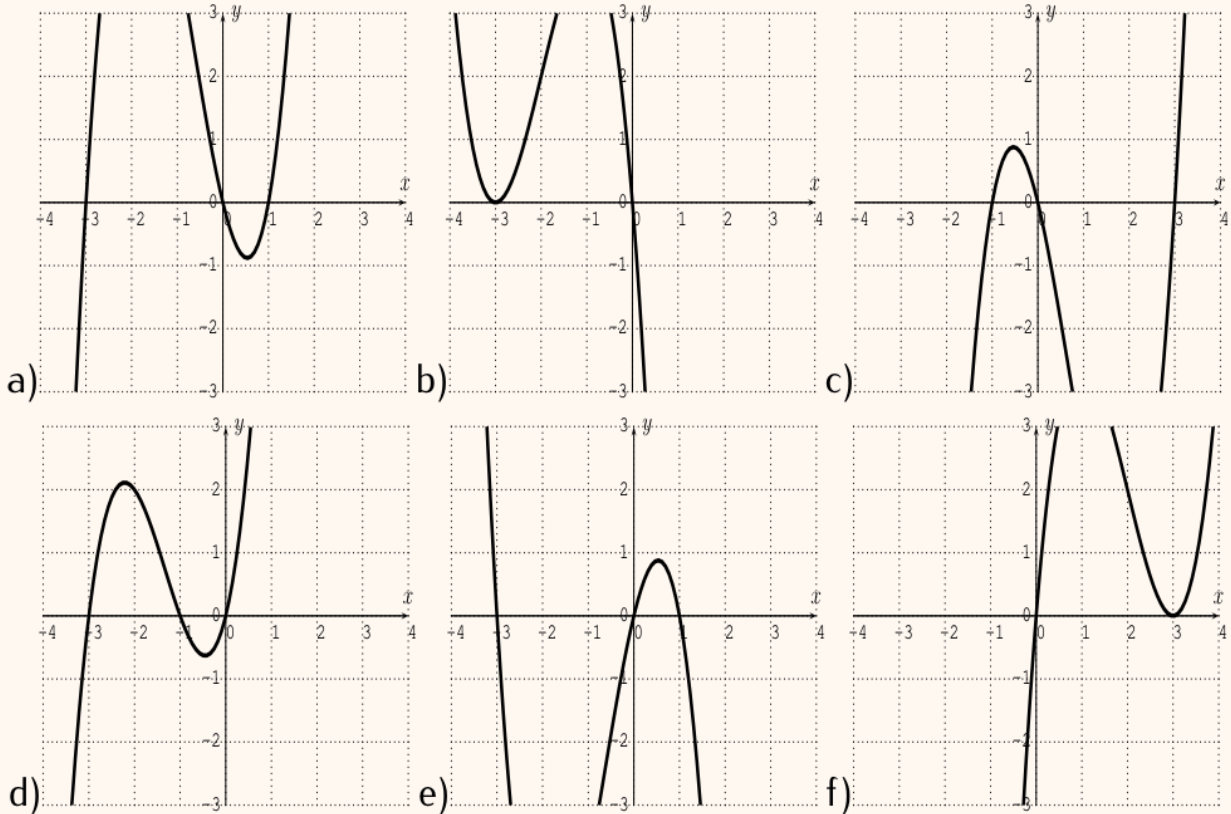
For each of the polynomials f , g , h , and k , find the corresponding graph from (a)-(f) below.

$$f(x) = x^3 + 4x^2 + 3x$$

$$g(x) = -x^3 - 2x^2 + 3x$$

$$h(x) = x^3 - 2x^2 - 3x$$

$$k(x) = -x^3 - 6x^2 - 9x$$



$$f(x) = x^3 + 4x^2 + 3x = x(x^2 + 4x + 3) = x(x+1)(x+3)$$

the roots of $f(x) \Rightarrow f(x) = 0 \Rightarrow x(x+1)(x+3) = 0$

$$\Rightarrow x=0, x=-1, x=-3$$

End behavior x^3 $\left\{ \begin{array}{l} \text{leading coeff} = 1 > 0 \\ \text{degree} = 3 (\text{odd}) \end{array} \right. \Rightarrow (\downarrow, \uparrow)$

$\Rightarrow$ graph (d)

$$g(x) = -x^3 - 2x^2 + 3x = -x(x^2 + 2x - 3) = -x(x+3)(x-1)$$

The root(s) of $g(x)$: $g(x) = -x(x-1)(x+3) = 0$

$$\Rightarrow -x=0, \quad x-1=0, \quad x+3=0$$

$$\Rightarrow x=0, \quad x=1, \quad x=-3$$

End behavior $-x^3$ $\left\{ \begin{array}{l} \text{leading coeff} = -1 < 0 \\ \text{degree} = 3 \text{ (odd)} \end{array} \right. \Rightarrow (\nearrow, \searrow)$

$\Rightarrow$ graph (e)

$$h(x) = x^3 - 2x^2 - 3x = x(x^2 - 2x - 3) = x(x+1)(x-3)$$

The roots of $h(x)$: $h(x) = x(x+1)(x-3) = 0$

$$\Rightarrow x=0 \text{ or } x+1=0 \text{ or } x-3=0$$

$$\Rightarrow x=0 \text{ or } x=-1 \text{ or } x=3$$

End behavior x^3 $\left\{ \begin{array}{l} \text{leading coeff} = 1 > 0 \\ \text{degree} = 3 \text{ (odd)} \end{array} \right. \Rightarrow (\swarrow, \nearrow)$

$\Rightarrow$ graph (c)

$$k(x) = -x^3 - 6x^2 - 9x = -x(x^2 + 6x + 9) = -x(x+3)^2$$

The roots of $k(x)$: $k(x) = -x(x+3)^2 = 0$

$$\Rightarrow x=0 \text{ or } x=-3, x=-3 \text{ (multiplicity=2)}$$

$\Rightarrow$ graph (b)

Exercise 8.5

Sketch a complete the graph of the function. Label all intercepts of the graph.

- ✓ a) $f(x) = x^3 + 4x^2 + x - 6$
- ✓ b) $f(x) = 2x^3 - 15x^2 + 34x - 24$
- ✓ c) $f(x) = x^3 - 16x - 21$

Sol
a) End behavior: leading term x^3 $\begin{cases} \text{leading coeff} = 1 > 0 \\ \text{degree} = 3 \text{ (odd)} \end{cases}$

$\Rightarrow (\downarrow, \uparrow)$

Roots (x-intercepts): factors of -6 (constant term) = $1, 2, 3, 6$
 $-1, -2, -3, -6$

education guess $x=1 \Rightarrow f(1) = 1^3 + 4 \cdot 1^2 + 1 - 6 = 0$
 $\Rightarrow x=1$ is a root

$\Rightarrow (x-1)$ is a factor of $f(x)$.

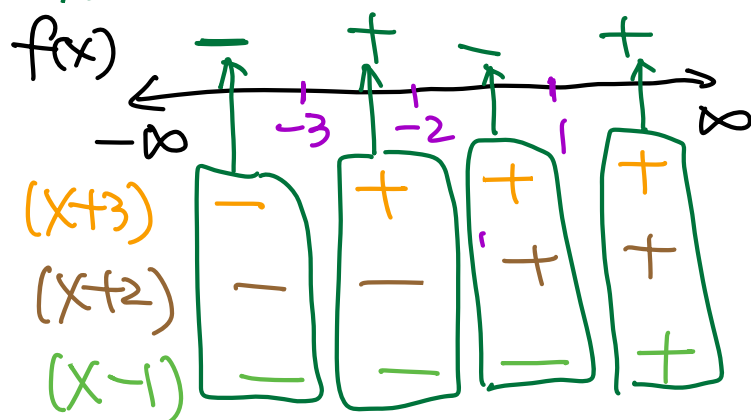
$$f(x) = (x-1) \cdot (x^2 + 5x + 6)$$

$$= (x-1)(x+2)(x+3)$$

$$\begin{array}{r} x^2 + 5x + 6 \\ x-1 \overline{) x^3 + 4x^2 + x - 6} \\ \underline{-(x^3 - x^2)} \\ 5x^2 + x \\ \underline{-(5x^2 - 5x)} \\ 6x - 6 \\ \underline{-(6x - 6)} \\ 0 \end{array}$$

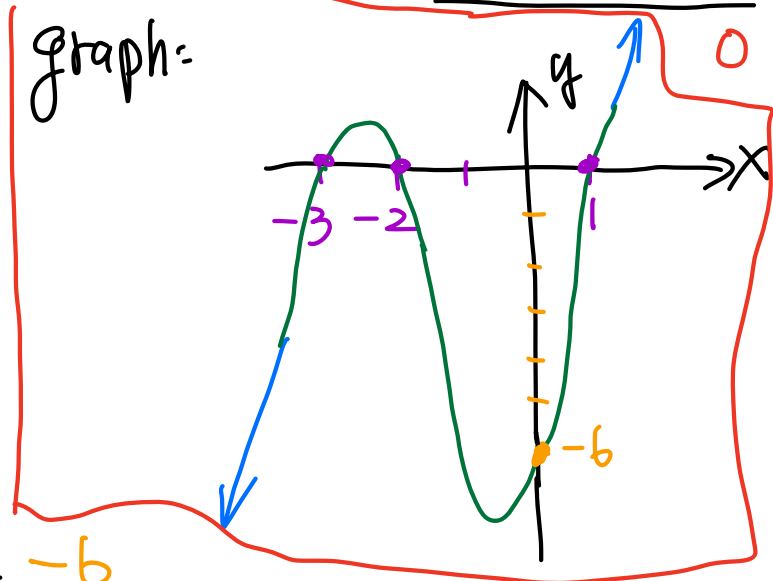
$f(x)$ has 3 roots: $x=1, x=-2, x=-3$

Number line:



y-intercept: ($x=0$, find $f(0)$)
 $f(0) = 0^3 + 4 \cdot 0^2 + 0 - 6 = -6$

Graph:



$$b) f(x) = 2x^3 - 15x^2 + 34x - 24$$

i) End behavior: leading term = $2x^3$
 leading coeff = $2 > 0$
 degree = 3 (odd)
 $\Rightarrow (\downarrow, \uparrow)$

ii) Roots: factors of constant term -24 : $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \dots$
 education guess:

$$\textcircled{1} X=1 \Rightarrow f(1) = 2 \cdot 1 - 15 \cdot 1 + 34 \cdot 1 - 24 = -3 \neq 0 \Rightarrow \text{not a root}$$

$$\textcircled{2} X=-1 \Rightarrow f(-1) = 2 \cdot (-1)^3 - 15 \cdot (-1)^2 + 34 \cdot (-1) - 24 = -2 - 15 - 34 - 24 \neq 0 \Rightarrow \text{not a root.}$$

$$\textcircled{3} X=2 \Rightarrow f(2) = 2 \cdot 2^3 - 15 \cdot 2^2 + 34 \cdot 2 - 24 = 16 - 60 + 68 - 24 = 0 \Rightarrow X=2 \text{ is a root!}$$

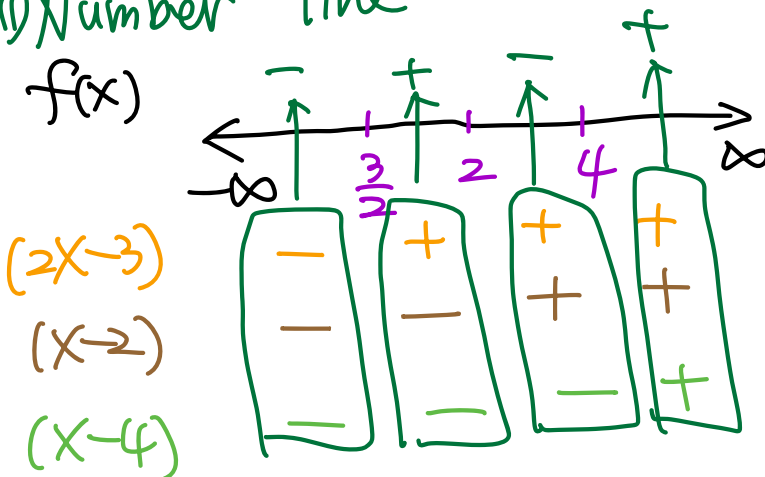
$$f(x) = (x-2)(2x^2 - 11x + 12)$$

$$= (x-2)(x-4)(2x-3)$$

$$x-2 \begin{array}{r} 2x^2 - 11x + 12 \\ 2x^3 - 15x^2 + 34x - 24 \\ -(2x^3 - 4x^2) \\ \hline 11x^2 + 34x \\ -(11x^2 + 22x) \\ \hline 12x - 24 \\ -(12x - 24) \\ \hline 0 \end{array}$$

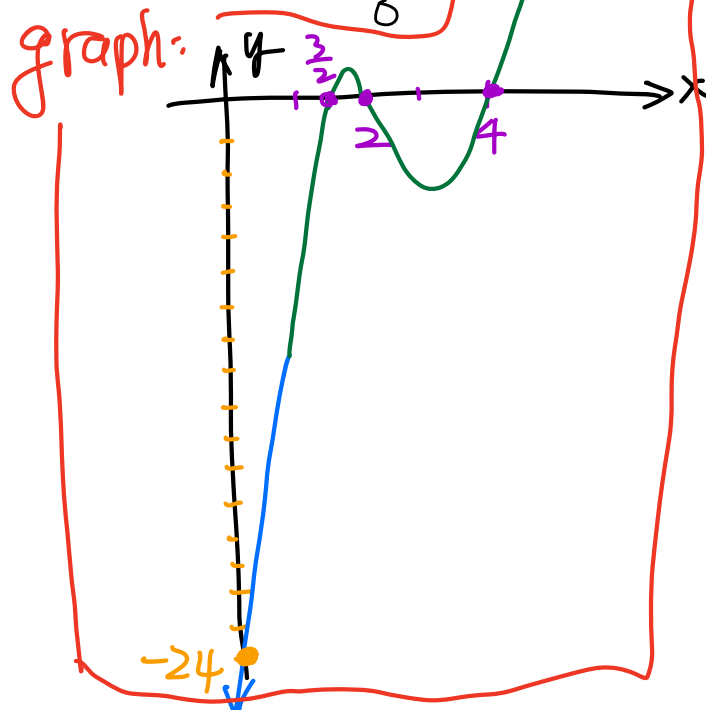
$f(x)$ has 3 roots: $x = \frac{3}{2}, 2, 4$

iii) Number line



iv) y-intercept:

$$f(0) = 2 \cdot 0^3 - 15 \cdot 0^2 + 34 \cdot 0 - 24 = -24$$



$$c) f(x) = x^3 - 16x - 21$$

i) End behavior: leading term $1x^3$ — leading coeff = $1 > 0 \Rightarrow (\downarrow, \uparrow)$
 — degree = 3 (odd)

ii) roots: factors of constant term " -21 ": $\pm 1, \pm 3, \pm 7, \pm 21$

education guess:

$$\textcircled{1} x=1, f(1) = 1^3 - 16 \cdot 1 - 21 \neq 0$$

$$\textcircled{2} x=-1, f(-1) = (-1)^3 - 16(-1) - 21 = -1 + 16 - 21 \neq 0$$

$$\textcircled{3} x=3, f(3) = 3^3 - 16 \cdot 3 - 21 \neq 0$$

$$\textcircled{4} x=-3, f(-3) = (-3)^3 - 16(-3) - 21 = -27 + 48 - 21 = 0$$

$\Rightarrow x = -3$ is a root.

$$f(x) = (x+3)(x^2 - 3x - 7) = 0$$

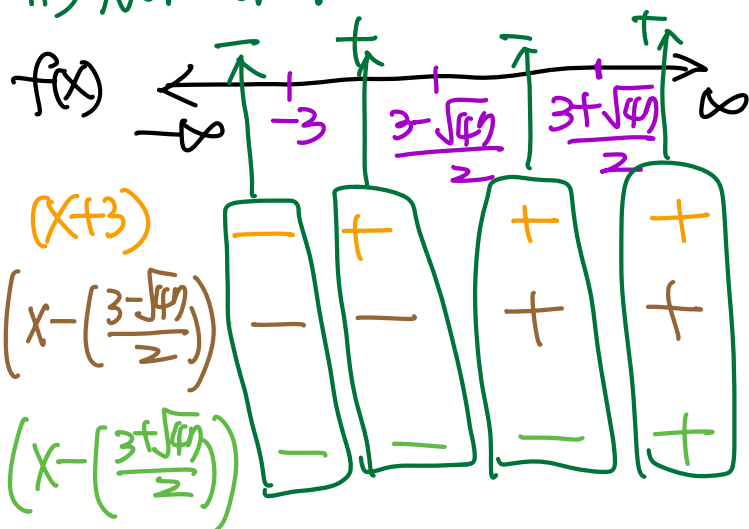
$$\begin{array}{r} x^2 - 3x - 7 \\ x+3 \overline{) \begin{array}{r} x^3 + 0x^2 - 16x - 21 \\ -(x^3 + 3x^2) \\ \hline -3x^2 - 16x \\ -(-3x^2 - 9x) \\ \hline -7x - 21 \\ -(-7x - 21) \\ \hline 0 \end{array}} \end{array}$$

$$\Rightarrow x+3=0 \text{ or } x^2 - 3x - 7 = 0$$

$$\Rightarrow x = -3 \text{ or } x = \frac{3 \pm \sqrt{9+28}}{2}$$

$$= \frac{3 \pm \sqrt{47}}{2}$$

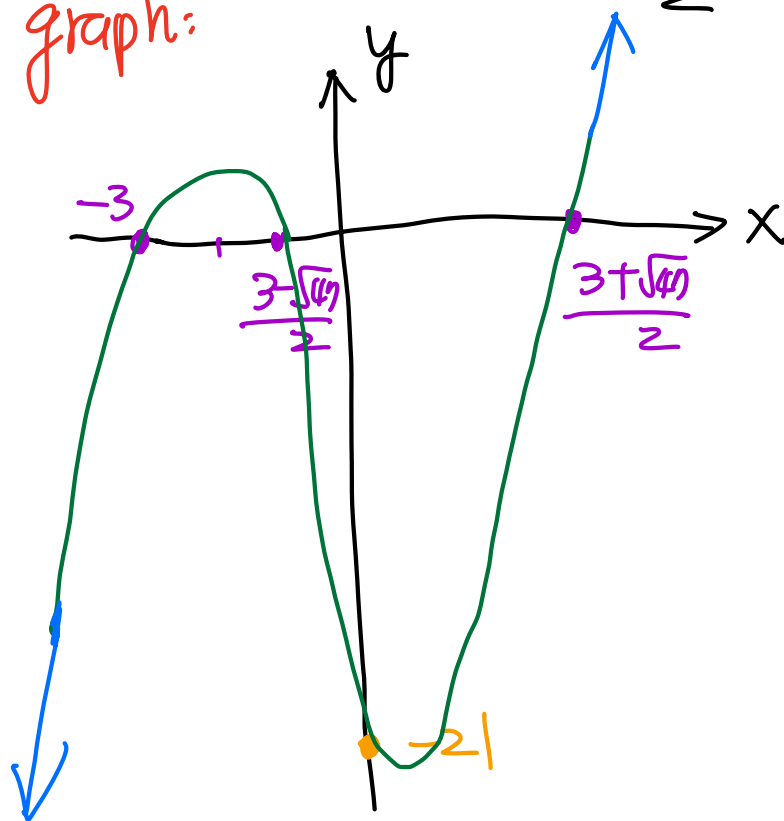
iii) Number line



iv) y-intercept

$$f(0) = 0^3 - 16 \cdot 0 - 21 = -21$$

graph:



Exercise 8.7

Find all roots and factor the polynomial completely.

- a) $f(x) = x^3 - 5x^2 + 2x + 8$
- ✓ b) $f(x) = x^3 + 7x^2 + 7x - 15$
- c) $f(x) = x^3 + 9x^2 + 26x + 24$
- d) $f(x) = x^3 + 4x^2 - 11x + 6$
- e) $f(x) = 3x^3 + 13x^2 - 52x + 28$
- f) $f(x) = 6x^3 - 5x^2 - 13x - 2$
- ✓ g) $f(x) = 6x^3 - x^2 - 31x - 10$
- h) $f(x) = x^3 - 7x^2 + 13x - 3$
- ✓ i) $f(x) = x^3 + 2x^2 - 11x + 8$
- ✓ j) $f(x) = 2x^3 + 7x^2 + 5x - 2$
- k) $f(x) = 3x^3 - 10x^2 - 4x + 21$

b) $f(x) = x^3 + 7x^2 + 7x - 15$,

education guess: the factor of constant term "-15"
 $\pm 1, \pm 3, \pm 5, \pm 15$

check which one is a root of $f(x)$.

① $x=1$, $f(1) = 1 + 7 + 7 - 15 = 0 \Rightarrow x=1$ is a root!

$$f(x) = (x-1) \cdot (x^2 + 8x + 15) = (x-1)(x+3)(x+5) = 0$$

$$\begin{array}{r} x-1 \overline{) \begin{array}{r} x^3 + 7x^2 + 7x - 15 \\ -(x^3 - x^2) \\ \hline 8x^2 + 7x \\ -(8x^2 - 8x) \\ \hline 15x - 15 \\ -(15x - 15) \\ \hline 0 \end{array}} \end{array}$$

$$\Rightarrow x=1 \text{ or } x=-3 \text{ or } x=-5$$

$$g) f(x) = 6x^3 - x^2 - 31x - 10$$

education guess: the factors of the constant term "-10"

$$\pm 1, \pm 2, \pm 5, \pm 10,$$

$$(\pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{5}{3}, \pm \frac{10}{3},$$

$$\pm \frac{1}{6}, \pm \frac{5}{6})$$

check roots

$$① x=1, f(1) = 6 \cdot 1^3 - 1^2 - 31 \cdot 1 - 10 \neq 0$$

$$② x=-1, f(-1) = 6(-1)^3 - (-1)^2 - 31(-1) - 10 \neq 0$$

$$③ x=2, f(2) = 6 \cdot 2^3 - 2^2 - 31 \cdot 2 - 10 \neq 0$$

$$④ \boxed{x=-2}, f(-2) = 6(-2)^3 - (-2)^2 - 31(-2) - 10 = -48 - 4 + 62 - 10 = 0 \text{ this is a root!}$$

$$f(x) = (x+2)(6x^2 - 13x - 5) = (x+2)(2x-5)(3x+1) = 0$$

$$\begin{array}{r} 6x^2 - 13x - 5 \\ x+2 \overline{) 6x^3 - x^2 - 31x - 10} \\ \underline{-(6x^3 + 12x^2)} \\ -13x^2 - 31x \\ \underline{-(-13x^2 - 26x)} \\ -5x - 10 \\ \underline{-(-5x - 10)} \\ 0 \end{array}$$

$$\boxed{x = -2 \text{ or } \frac{5}{2} \text{ or } -\frac{1}{3}}$$

$$i) f(x) = x^3 + 2x^2 - 11x + 8$$

education guess: the factors of 8:

$$\pm 1, \pm 2, \pm 4, \pm 8$$

check roots:

① $x=1$, $f(1)=1^3+2\cdot 1^2-11\cdot 1+8=0$ this is a root!

$$f(x) = (x-1)(x^2+3x-8) = 0$$

$$\Rightarrow x-1=0 \text{ or } x^2+3x-8=0$$

$$\Rightarrow x=1 \text{ or } x = \frac{-3 \pm \sqrt{9+32}}{2}$$

$$= \frac{-3 \pm \sqrt{41}}{2}$$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -11 & 8 \\ & & 1 & 3 & -8 \\ \hline & 1 & 3 & -8 & 0 \end{array}$$

$$\Rightarrow x^2+3x-8$$

j) $f(x) = 2x^3+7x^2+5x-2$

education guess: the factor of "-2"
 $\pm 1, \pm 2, (\pm \frac{1}{2})$

check roots:

① $x=1$, $f(1)=2\cdot 1+7\cdot 1+5\cdot 1-2 \neq 0$

② $x=-1$, $f(-1)=2\cdot (-1)^3+7\cdot (-1)^2+5(-1)-2$
 $= -2+7-5-2 \neq 0$

③ $x=2$, $f(2)=2\cdot 2^3+7\cdot 2^2+5\cdot 2-2 \neq 0$

④ $x=-2$, $f(-2)=2\cdot (-2)^3+7\cdot (-2)^2+5(-2)-2$
 $= -16+28-10-2=0$

this is a root!

$$f(x) = (x+2)(2x^2+3x-1) = 0$$

$$\begin{array}{r|rrrr} -2 & 2 & 7 & 5 & -2 \\ & & -4 & -6 & +2 \\ \hline & 2 & 3 & -1 & 0 \end{array}$$

$$\underline{2x^2+3x-1}$$

$$x+2=0 \text{ or } 2x^2+3x-1=0$$

$$\Rightarrow x=-2 \text{ or } x = \frac{-3 \pm \sqrt{9+8}}{4}$$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

Exercise 8.8

Graph the following polynomials without using the calculator.

✓ a) $f(x) = (x+4)^2(x-5)$

End behavior: $f(x) = (x+4)(x+4)(x-5)$

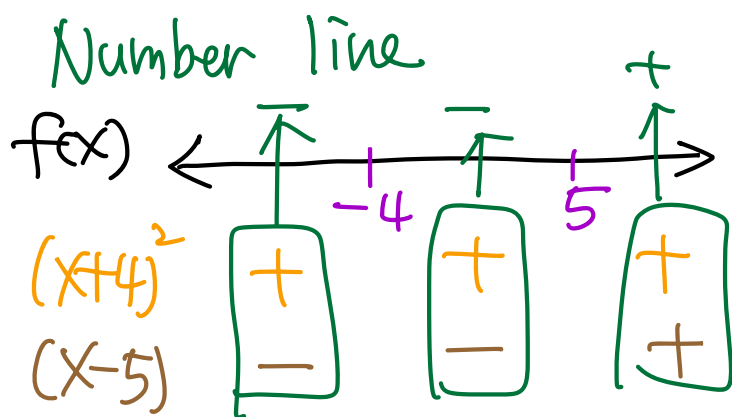
leading term $= \overbrace{x} \cdot \overbrace{x} \cdot \overbrace{x} = x^3 \Rightarrow (\downarrow, \uparrow)$

Roots: $f(x) = (x+4)^2(x-5) = 0$

$x+4=0$ or $x+4=0$ or $x-5=0$

$\Rightarrow x = -4$ or $x = -4$ or $x = 5$

y-intercept: $f(0) = (0+4)^2(0-5)$
 $= 16 \cdot (-5) = -80$



graph:

