MAT1375, Classwork19, Fall2026

Ch21. Trigonometric Identities

1. Addition and Subtraction of angles formulas:

Let α, β be two angles. We have

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

(1) $\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$

(2) $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$

(3) $\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$

(4) $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$

2. Half- and double-angle formulas:

(5) $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ (From (1) and let $\beta = \alpha$)

(6) $\cos(2\alpha) = \cos^2(\alpha) - \sin^2(\alpha)$ (From (3) and let $\beta = \alpha$)

(7) $\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$

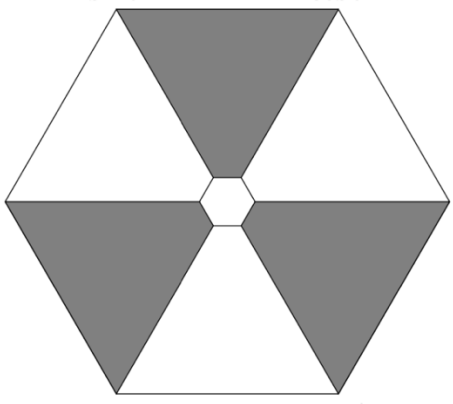
(8) $\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

3. Pythagorean Identities:

(9) $\sin^2(\theta) + \cos^2(\theta) = 1^2$

(10) $1^2 + \tan^2(\theta) = \sec^2(\theta)$

(11) $1^2 + \cot^2(\theta) = \csc^2(\theta)$



4. Find the exact value of the trigonometric functions:

a) $\sin\left(\frac{11\pi}{12}\right)$ b) $\cos\left(\frac{7\pi}{8}\right)$ c) $\sin(15°)$ d) $\cos(75°)$

$$\sin\left(\frac{3\pi}{12}+\frac{8\pi}{12}\right) = \sin\left(\frac{3\pi}{12}\right)\cdot\cos\left(\frac{8\pi}{12}\right) + \cos\left(\frac{3\pi}{12}\right)\cdot\sin\left(\frac{8\pi}{12}\right)$$
$$= \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{2\pi}{3}\right) + \cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{2\pi}{3}\right)$$

5. Simplify the given function using the addition and subtraction formulas.

a) $\sin\left(\frac{\pi}{2}+x\right)$ b) $\sin\left(\frac{\pi}{2}-x\right)$ c) $\cos\left(\frac{\pi}{2}+x\right)$ d) $\cos\left(\frac{\pi}{2}-x\right)$