MAT1375, Classwork16, Fall2026

Ch15. Exponential Equations and Applications

1. The Exponential and Logarithmic functions and one-to-one property:

For $b > 0, b \neq 1$, the exponential and logarithmic functions are one-to-one:

$b^x = b^y \Leftrightarrow$ $x = y$ (same output)

$\log_b(x) = \log_b(y) \Leftrightarrow$ $x = y$ (same input.)

2. Solve for x:

(a) $2^{x+7} = 32$.

$2^{x+7} = 2^5$

one-to-one property $\Rightarrow x+7 = 5$

$x = -2$

(b) $10^{2x-8} = 0.01$.

$0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

$10^{2x-8} = 10^{-2}$

1-1 $\Rightarrow 2x-8 = -2$

$x = 3$

(c) $27^{x+3} = 9^{x-1}$.

$27 = 3^3$; $9 = 3^2$

$(3^3)^{(x+3)} = (3^2)^{(x-1)}$

$3^{3(x+3)} = 3^{2(x-1)}$

1-1 $\Rightarrow 3(x+3) = 2(x-1)$

$3x+9 = 2x-2$

$\Rightarrow x = -11$

(d) $8^{x+2} = 4^{x-3}$.

$8 = 2^3$; $4 = 2^2$

$2^{3(x+2)} = 2^{2(x-3)}$

1-1 $\Rightarrow 3(x+2) = 2(x-3)$

$3x+6 = 2x-6$

$x = -12$.

3. How about the equations **without** the same base on both sides? (For example, $4^x = 15$.)

$4^x = 15 \Rightarrow x = \log_4(15)$

4. Using **Logarithms** to Solve **Exponential Equations**:

Step1: Isolate the exponential expression.

Step2: Take logarithm on both sides of the equation for common base 10 or natural base e.

($\log_{10}()$ $\ln()$)

Step3: Simplify using one of these properties:

$\ln(b^x) =$ $x \cdot \ln(b)$ (power rule) if b is not 10 *or* $\ln(e^x) =$ $x \cdot 1 = x$ *or* $\log(10^x) =$ $x \cdot \log(10) = x$.

($x \cdot \ln(e)$)

Step4: Solve for x.

5. Solve for x:

$3^{x+5} = 8$.

Step1. Done

Step2 Take "ln" on both sides

$\ln(3^{x+5}) = \ln(8)$

power rule $(x+5) \cdot \ln(3) = \ln(8)$

$x \cdot \ln(3) + 5 \cdot \ln(3) = \ln(8)$

$-5\ln(3) \quad -5\ln(3)$

$x \cdot \ln(3) = \ln(8) - 5 \cdot \ln(3)$

$\ln(3)$ $\ln(3)$

$x = \frac{\ln(8) - 5\ln(3)}{\ln(3)}$

6. Solve for x.

(a) $13^{2x-4} = 6$. (b) $5.1^{x} = 2.7^{2x+6}$. (c) $7e^{2x} - 5 = 58$. (d) $10^{x} = 800$.

$\log_{10}(\ )$

(a) Take "ln" on both sides

$\ln(13^{2x-4}) = \ln(6)$

power rule

$(2x-4)\cdot\ln(13) = \ln(6)$

$x\cdot 2\ln(13) - 4\ln(13) = \ln(6)$

$+4\ln(6) \quad +4\ln(13)$

$x\cdot 2\ln(13) = \ln(6) + 4\cdot\ln(13)$

$$x = \frac{\ln(6) + 4\cdot\ln(13)}{2\cdot\ln(13)}$$

(c) Isolation: $7e^{2x} - 5 = 58$

$+5 \quad +5$

$\Rightarrow 7e^{2x} = 63$

$\Rightarrow e^{2x} = \frac{63}{7} \Rightarrow e^{2x} = 9$

Take "ln" on both sides

$\ln(e^{2x}) = \ln(9)$

power rule

$2x\cdot\ln(e) = \ln(9)$

$\ln(e) = 1, \Rightarrow 2x = \ln(9)$

$$x = \frac{\ln(9)}{2}$$

(d)

$\log_{10}(10^{x}) = \log_{10}(800)$

power rule

$x\cdot\log_{10}(10) = \log_{10}(800)$

Since $\log_{10}(10) = 1$, then $x = \log_{10}(800)$

$$x = \frac{\ln(800)}{\ln(10)}$$

(b) $\ln(5.1^{x}) = \ln(2.7^{2x+6})$

power rule

$x\cdot\ln(5.1) = (2x+6)\cdot\ln(2.7)$

$x\cdot\ln(5.1) = x\cdot 2\ln(2.7) + 6\cdot\ln(2.7)$

$x\cdot\ln(5.1) - x\cdot 2\ln(2.7) = 6\cdot\ln(2.7)$

$x\cdot(\ln(5.1) - 2\ln(2.7)) = 6\cdot\ln(2.7)$

$$x = \frac{6\cdot\ln(2.7)}{\ln(5.1) - 2\cdot\ln(2.7)}$$