

Ch12. Solving inequalities

1. The Strategy for Solving Inequalities (Application of **Number Line Test**):

Step1. Replace ' $>$ ' (' $\geq$ ') or ' $<$ ' (' $\leq$ ') by ' $=$ ' and solve the equation.

Step2. Mark the solutions on the number line and check sign (positive/negative) in each subinterval.

Step3. Check the end points of the subintervals to see if they are included in the solution set.

2. Given $x^3 + 15x \geq 7x^2 + 9$. Solve for x .
 move it to make the right hand side be zero

① $x^3 + 15x - 7x^2 - 9 \geq 0$

② Replace " $\geq$ " with " $=$ " $f(x) = x^3 - 7x^2 + 15x - 9 = 0$

③ Find the roots of $f(x)$ (the guess of roots is the factor of -9 :
 $f(1) = 1^3 + 15 \cdot 1 - 7 \cdot 1^2 - 9 = 16 - 16 = 0 \Rightarrow$ zero of $f(x)$ is 1
 $\Rightarrow (x-1)$ is a factor of $f(x)$

Polynomial division:

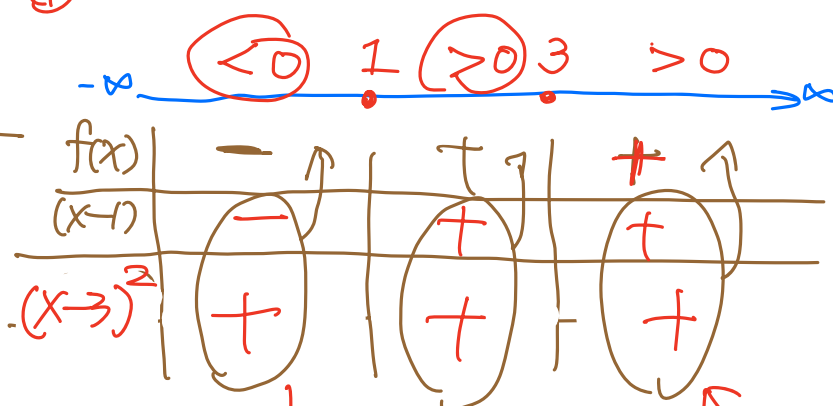
$$\begin{array}{r} x^2 - 6x + 9 \\ x \overline{) x^3 - 7x^2 + 15x - 9} \\ \underline{x^3 - x^2} \\ -6x^2 + 15x \\ \underline{-6x^2 + 6x} \\ 9x - 9 \\ \underline{9x - 9} \\ 0 \end{array}$$

$\frac{x^3}{x} = x^2$
 $x^2(x-1)$
 $\frac{-6x^2}{x} = -6x$
 $-6x(x-1)$
 $\frac{9x}{x} = 9$
 $9(x-1)$

$f(x) = (x-1)(x^2 - 6x + 9)$
 $= (x-1)(x-3)(x-3) = 0$

zeros of $f(x)$ $(x-1)=0, (x-3)=0, (x-3)=0$
 roots $x=1$ or $x=3$ or $x=3$

④ Number line test



⑤ $f(x) = x^3 - 7x^2 + 15x - 9 \geq 0$
 $x \in (1, 3) \cup [3, \infty)$

check the endpoints:

$x=1, f(1)=0$ included
 $x=3, f(3)=0$ included

If $x \in [1, \infty)$, then $f(x) \geq 0$
 (inequality)

3. Solve for x : $|2x-3| \geq 7$.

① $|2x-3| = 7$ (Replace " $\geq$ " with " $=$ ")

② Get rid of the absolute value sign:

$2x-3 = +7$ or $2x-3 = -7$

$2x = 7+3$ or $2x = -7+3$

$x = 5$

$x = -2$



$|2x-3|-7$

$|2(-2)-3|-7 = -2 > 0$

$|2(5)-3|-7 = 2 > 0$

$|2(6)-3|-7 = 2 > 0$

check the endpoints,

$x = -2,$

$|2(-2)-3|-7 = 0$

included

$x = 5$

$|2(5)-3|-7 = 0$ included

If $x \in (-\infty, -2] \cup [5, \infty)$, then

4. Solve for x : $\frac{x^2-5x+6}{x^2-5x} \geq 0$.

① Replace " $\geq$ " with " $=$ " $f(x) = \frac{x^2-5x+6}{x^2-5x} = 0$, $\frac{x^2-5x+6}{x^2-5x} = \frac{(x-2)(x-3)}{x(x-5)}$

② x -intercept (when $f(x)=0$, $x=?$) $\rightarrow \frac{(x-2)(x-3)}{x(x-5)} = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x=2$ or $x=3$

(V.A) Vertical Asymptotes

(H.A) Horizontal Asymptotes

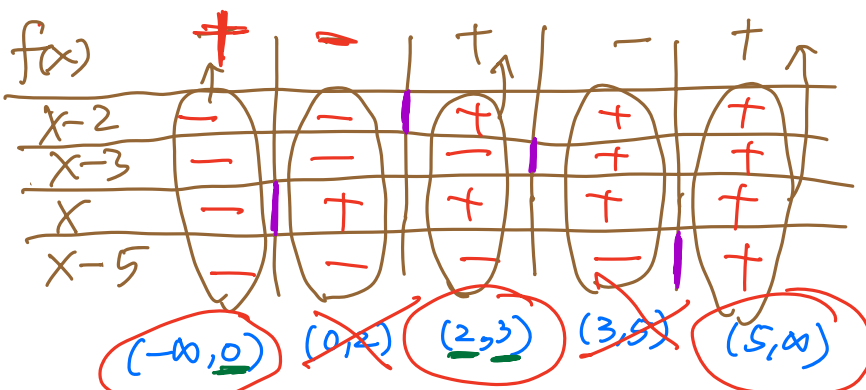
y -intercept (when $x=0$, $f(x)=?$)

NO y -intercept.

The line $x=0$ or $x=5$ is V.A.

$y = \frac{1}{1} = 1$ (deg $(x^2-5x+6) = 2$, deg $(x^2-5x) = 2$) $\Rightarrow$ H.A $y = \text{ratio of leading coefficients}$

③ Number line test



$f(x) = \frac{x^2-5x+6}{x^2-5x} \geq 0$ $f(x) = \frac{(x-2)(x-3)}{x(x-5)}$

$x \in (-\infty, 0) \cup [2, 3] \cup (5, \infty)$

If $x \in (-\infty, 0) \cup [2, 3] \cup (5, \infty)$

Then $f(x) = \frac{x^2-5x+6}{x^2-5x} \geq 0$

Check endpoints

$x=0$ $f(0) = \frac{6}{0}$ undefined NOT included

$x=2$ $f(2) = \frac{0}{-6} = 0$ included

$x=3$ $f(3) = \frac{0}{-6} = 0$ included

$x=5$ $f(5) = \frac{6}{0}$ undefined NOT included