

MAT1375, Classwork11, Fall2026

Ch10. Rational Functions II

6. The order of growth for functions: **Power functions.**

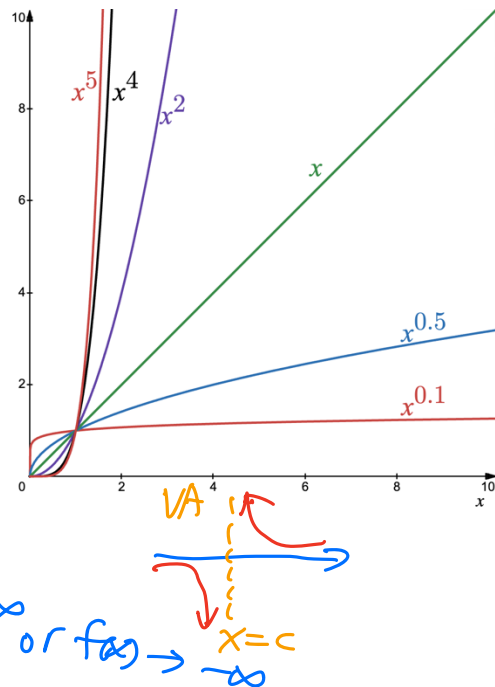
$$x^p, p > 0$$

For each $x > 1$, we have

$$\dots < x^{0.1} < x^{0.5} < x < x^2 < x^4 < x^5 < \dots$$

7. The definition of a **Horizontal Asymptote**:

The line $y = b$ is a horizontal asymptote of the graph of a function f if $f(x)$ approaches b as x increases or decreases without bound.



8. What is the difference of Vertical Asymptote and Horizontal Asymptote?

Vertical Asymptote $x = c$ when $x \rightarrow c$ $f(x) \rightarrow \infty$ or $f(x) \rightarrow -\infty$

Horizontal Asymptote $y = \frac{p}{q}$ when $x \rightarrow \infty$ or $x \rightarrow -\infty$, $f(x) \rightarrow \frac{p}{q}$

9. How to locate Horizontal Asymptotes: Let $f(x) = \frac{p(x)}{q(x)}$ be a rational function given by

$$f(x) = \frac{p_n x^n + p_{n-1} x^{n-1} + \dots + p_1 x + p_0}{q_m x^m + q_{m-1} x^{m-1} + \dots + q_1 x + q_0}, p_n \neq 0, q_m \neq 0$$

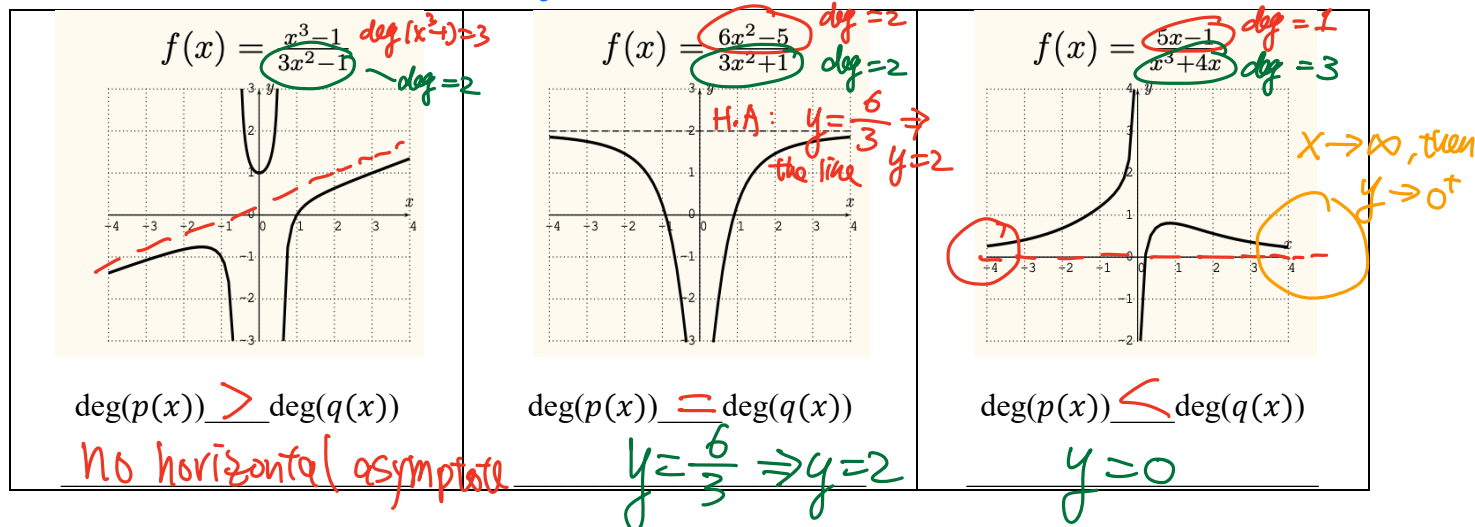
deg n deg m

where the degree of the numerator is n and the degree of the denominator is m.

(1) If $n > m$, the graph of f has NO horizontal asymptote.

(2) If $n = m$, the line $y = \frac{p_n}{q_m}$ (which is the ratio of two leading coefficients) is the horizontal asymptote of the graph of f .

(3) If $n < m$, the X-axis (which is $y = 0$) is the horizontal asymptote of the graph of f .



Ch11. More on Rational Functions

1. The graph of $f(x) = \frac{p(x)}{g(x)}$ is displayed below, where $\deg(p(x))=1$ and $\deg(g(x))=3$.

Find the intercepts, asymptotes, and a formula for $f(x)$.

✓ X-intercept $(2, 0)$

$$0 = f(2) = \frac{p(2)}{g(2)} \Rightarrow 0 \cdot g(2) = \frac{p(2)}{g(2)} \cdot g(2)$$

→ 2 is a root of $p(x)$.

$(x-2)$ is a factor of $p(x)$, $p(x) = A \cdot (x-2)$

Y-intercept $(0, -1)$

$$x=0, f(0) = -1$$

✓ Vertical Asym (V.A)

$$x=-3, x=1, x=4$$

$$x+3=0 \quad x-1=0 \quad x-4=0$$

$$(x+3)(x-1)(x-4)=0 \Rightarrow g(x) = (x+3)(x-1)(x-4)$$

X Horizontal Asym (H.A)

$$y=0$$

$$f(x) = \frac{A(x-2)}{(x+3)(x-1)(x-4)} = \frac{6(x-2)}{(x+3)(x-1)(x-4)}$$

$$-1 = f(0) = \frac{A(0-2)}{(0+3)(0-1)(0-4)} = \frac{-2A}{12} \Rightarrow -1 = \frac{-2A}{12}$$

For 2. And 3., let $f(x) = \frac{p(x)}{g(x)}$ be a rational function and $\deg(p(x)) > \deg(g(x))$.

2. Rational Function and Long Division:

If $\frac{\text{dividen}}{p(x)}$ divided by $\frac{\text{divisor}}{g(x)}$ can be represented with a quotient $q(x)$ and a remainder $r(x)$ where

$\deg(r(x)) < \deg(g(x))$, one can rewrite $f(x)$ as

$$\frac{p(x)}{g(x)} = \frac{g(x) \cdot q(x) + r(x)}{g(x)}$$

$$f(x) = \frac{p(x)}{g(x)} = \frac{g(x)}{g(x)} \cdot q(x) + \frac{r(x)}{g(x)}$$

$$\frac{g(x) \cdot q(x)}{g(x)} + \frac{r(x)}{g(x)}$$

3. Asymptotic Behavior with Slant Asymptote:

Since $\deg(r(x)) < \deg(g(x))$, for large $|x|$ (which is $x \rightarrow \pm \infty$), we have

$\frac{r(x)}{g(x)}$ approaches zero so that $f(x) \rightarrow q(x)$.

If $q(x)$ is a linear degree 1 polynomial (which is a polynomial of degree 1), then q is called the slant asymptote of f .

4. Find the **slant asymptote** of the rational function $f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6}$.

$$\begin{array}{r} \textcircled{2x^3} - 13x^2 + 35x - 26 \\ \underline{-(2x^3 - 8x^2 + 12x)} \\ -5x^2 + 23x - 26 \\ \underline{-(-5x^2 + 20x - 30)} \\ 3x + 4 \end{array}$$

$$\frac{2x^3}{x^2} = 2x$$

$$2x(x^2 - 4x + 6)$$

$$\frac{-5x^2}{x^2} = -5$$

$$-5(x^2 - 4x + 6)$$

$$f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6} = \frac{\text{dividen}}{\text{divisor}} = 2x - 5 + \frac{3x + 4}{x^2 - 4x + 6}$$

the slant asymptote of $f(x)$ is $y = 2x - 5$

