

MAT1375, Classwork10, Fall2026

Ch10. Rational Functions I

1. Definition of the Rational function:

A rational function is a fraction of two polynomials $f(x) = \frac{p(x)}{g(x)}$, where $p(x)$ and $g(x)$ are both polynomials, and $g(x) \neq 0$.

The **domain of a rational function** f is all real numbers for which the denominator $g(x)$ is not zero:

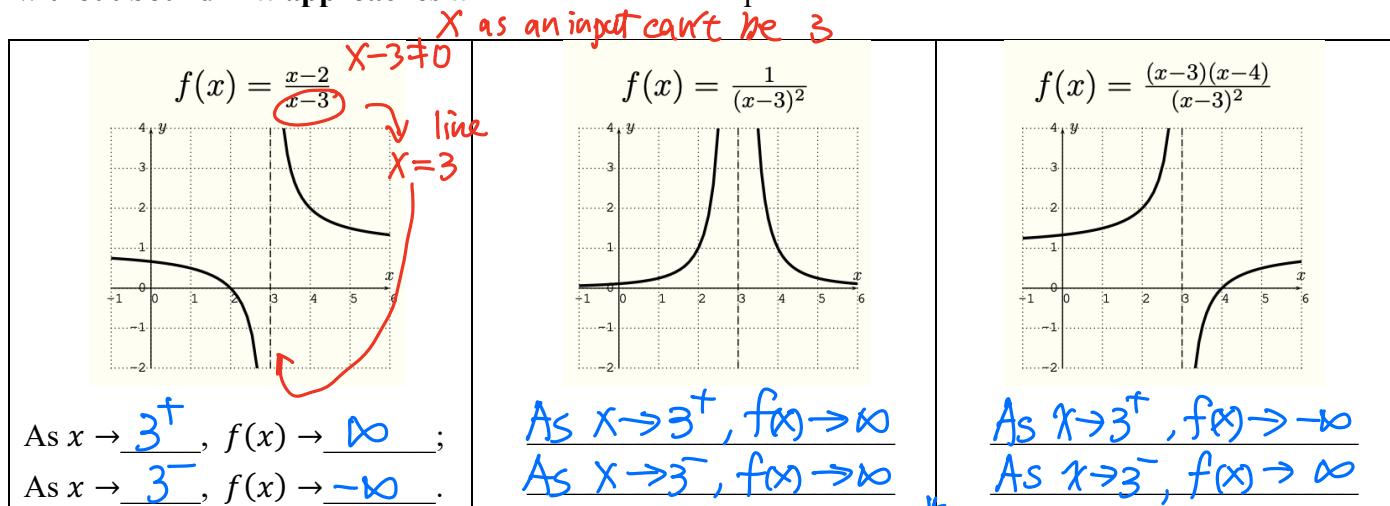
$$D_f = \{x \in \mathbb{R} \mid x \in D_p \cap D_g, \text{ but } g(x) \neq 0\}$$

2. Arrow Notation: Given a constant a and we have

$x \rightarrow a^+$:	x approaches a from the right (x is very closed to a but $x \neq a$ and $x > a$)
$x \rightarrow a^-$:	x approaches a from the left (x is very closed to a but $x \neq a$ and $x < a$)
$x \rightarrow \infty$:	x approaches infinity (x increases without bound)
$x \rightarrow -\infty$:	x approaches negative infinity (x decreases without bound)

3. The definition of a Vertical Asymptote:

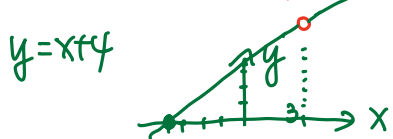
The line $x = a$ is a Vertical Asymptote of the graph of a function f if $f(x)$ **increases or decreases without bound** as x approaches a . Here are three examples:



4. How to locate Vertical Asymptotes: Let $f(x) = \frac{p(x)}{g(x)}$ be a rational function.

(1) If $p(x)$ and $g(x)$ have no common factors, and a is a zero of $g(x)$ which makes $f(x)$ undefined, then the line $x=a$ is a vertical asymptote of the graph of $f(x)$.

(2) If a is a root of both $p(x)$ and $g(x)$ ($p(a) = 0$, $g(a) = 0$) which means $x-a$ is the common factor of $p(x)$ and $g(x)$, then there is a jump / removable discontinuity at $x=a$ and there is no vertical asymptote at $x=a$.



$$y = \frac{(x-3)(x+4)}{(x-3)} = (x+4)$$

assume $x-3 \neq 0$
input $x \neq 3$
with $x \neq 3$

5. Find the vertical asymptotes of the graph of each rational function:

a) $f(x) = \frac{x}{x^2-1}$ $\deg 1$ $\deg 2$ $1 < 2$ b) $g(x) = \frac{x-1}{x^2-1}$ $\rightarrow$ factorize $x^2-1 = (x+1)(x-1) = 0$ c) $h(x) = \frac{x-1}{x^2+1}$ $\frac{x-1 \geq 0 \text{ or } x-1 < 0}{x^2+1 \geq 0 \text{ or } x^2+1 > 0}$

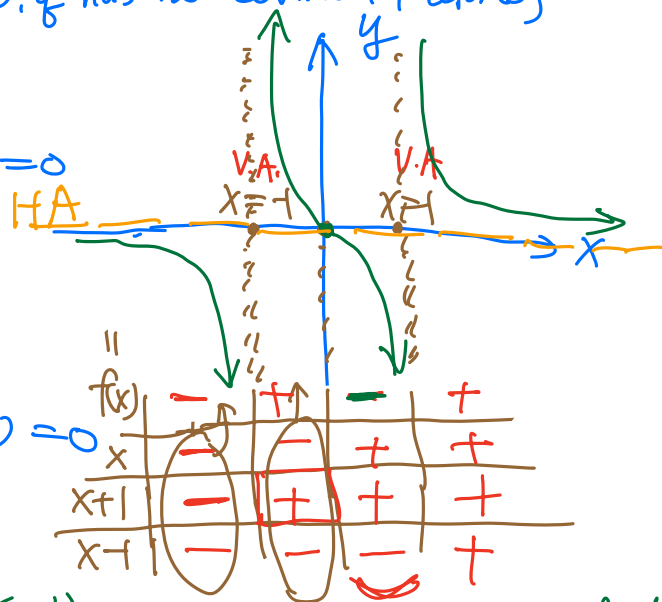
a) $f(x) = \frac{x}{(x+1)(x-1)}$ $p(x)=x$, $q(x)=(x+1)(x-1)$, p, q has no common factors

To find vertical asymptotes, let $q(x)=0$

$q(x)=(x+1)(x-1)=0 \Rightarrow x+1=0 \text{ or } x-1=0$

It means that the line $x=-1$ and $x=1$ are the vertical asymptotes (V.A)

A root of $f(x)$ as an input will make $f(x)=0$
 $x=0$ is a root of $f(x)$



b) $f(x) = \frac{x-1}{(x+1)(x-1)}$, $p(x)=(x-1)$, $q(x)=(x+1)(x-1)$ and they have common factor

$= \frac{1}{x+1}$ with $x \neq -1$

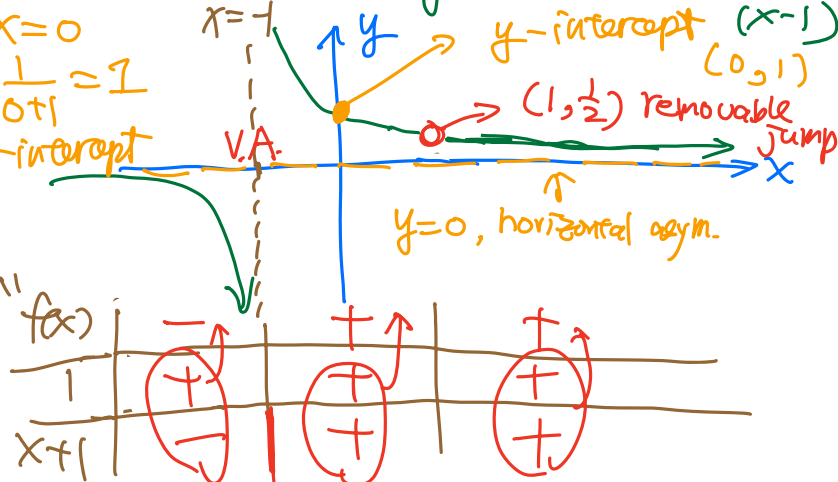
To find vertical asymptotes,

it means $x+1=0$

it implies the line $x=-1$ is a vertical asymptote.

where $x=0$
 $y = \frac{1}{0+1} = 1$
y-intercept

$\frac{1}{x+1}$



c) $h(x) = \frac{x-1}{x^2+1}$. $x^2+1 \geq 1 \Rightarrow$ it can never be zero $\Rightarrow$ No vertical asymptote.

X-intercept: $\frac{x-1}{x^2+1} = 0 \Rightarrow (x-1)=0 \Rightarrow x=1$, root is 1

$y=0$ X-intercept is (1,0)

y-intercept $y=h(0) = \frac{0-1}{0^2+1} = \frac{-1}{1} = -1$

horizontal asymptotes: $\deg(x-1)=1 < \deg(x^2+1)=2$
 $\Rightarrow y=0$ is a H.A.

