Mat 1575 HW 19 (§6.3)

In the following exercises, find the Taylor polynomials of degree two approximating the given function centered at the given point.

118. $f(x) = \cos(2x)$ at $a = \pi$

$f'(x) = -2\sin(2x)$, $f''(x) = -4\cos(2x)$

$P_2(x) = f(\pi) + \frac{f'(\pi)}{1!}(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2$

$= \cos(2\pi) - 2\sin(2\pi)(x-\pi) - 4\frac{\cos(2\pi)}{2!}(x-\pi)^2$ ($\sin(2\pi)=0$, $\cos(2\pi)=1$)

$= \boxed{1 - 2(x-\pi)^2}$

119. $f(x) = \sin(2x)$ at $a = \frac{\pi}{2}$

$f'(x) = 2\cos(2x)$, $f''(x) = -4\sin(2x)$

$P_2(x) = f(\frac{\pi}{2}) + \frac{f'(\frac{\pi}{2})}{1!}(x-\frac{\pi}{2}) + \frac{f''(\frac{\pi}{2})}{2!}(x-\frac{\pi}{2})^2$

$= \sin(2\cdot\frac{\pi}{2}) + \frac{2\cos(2\cdot\frac{\pi}{2})}{1!}(x-\frac{\pi}{2}) + \frac{-4\sin(2\cdot\frac{\pi}{2})}{2!}(x-\frac{\pi}{2})^2$ ($\sin(2\cdot\frac{\pi}{2})=0$, $\cos(2\cdot\frac{\pi}{2})=-1$, $-4\sin(2\cdot\frac{\pi}{2})=0$)

$= -2(x-\frac{\pi}{2})$

120. $f(x) = \sqrt{x}$ at $a = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$, $f''(x) = \frac{1}{2}\cdot(-\frac{1}{2})\frac{1}{(\sqrt{x})^3} = \frac{1}{-4(\sqrt{x})^3}$

$P_2(x) = f(4) + \frac{f'(4)}{1!}(x-4) + \frac{f''(4)}{2!}(x-4)^2$

$= \sqrt{4} + \frac{1}{2\cdot\sqrt{4}}(x-4) + \frac{1}{2}\cdot\left(\frac{1}{-4(\sqrt{4})^3}\right)(x-4)^2$

$= 2 + \frac{x-4}{4} - \frac{(x-4)^2}{64}$

121. $f(x) = \ln x$ at $a = 1$

$f'(x) = \frac{1}{x}$, $f''(x) = \frac{-1}{x^2}$

$P_2(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$= \ln 1 + \frac{1}{1}(x-1) + \frac{-1}{2}(x-1)^2 = (x-1) - \frac{(x-1)^2}{2}$ ($\ln 1 = 0$)

122. $f(x) = \frac{1}{x}$ at $a = 1$

$f'(x) = \frac{-1}{x^2}$ $f''(x) = \frac{2}{x^3}$

$P_2(x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2$

$= \frac{1}{1} + \frac{-1}{1!}(x-1) + \frac{2}{2!}(x-1)^2 = 1 - (x-1) + (x-1)^2$

123. $f(x) = e^x$ at $a = 1$ $f'(x)=e^x$, $f''(x)=e^x$

$P_2(x)=f(1)+\frac{f'(1)}{1!}(x-1)+\frac{f''(1)}{2!}(x-1)^2$

$= e^1 + \frac{e^1}{1!}(x-1) + \frac{e^1}{2!}(x-1)^2 = e + e(x-1)+\frac{e}{2}(x-1)^2$