

Mat 1575 HW 7 (§3.2)

Evaluate each of the following integrals by u -substitution.

73. $\int \sin^3 x \cos x \, dx$ $u = \sin x$
 $du = \cos x \, dx$
 $= \int u^3 \, du = \frac{u^4}{4} + C$
 $= \frac{\sin^4 x}{4} + C$

74. $\int \sqrt{\cos x} \sin x \, dx$ $u = \cos x, du = -\sin x \, dx$
 $-du = \sin x \, dx$
 $= \int -\sqrt{u} \, du = -\frac{2}{3} u^{\frac{3}{2}} + C$
 $= -\frac{2}{3} \sqrt{\cos^3 x} + C$

78. $\int \tan^2 x \sec^2 x \, dx$ $u = \tan x$
 $du = \sec^2 x \, dx$
 $= \int u^2 \, du = \frac{u^3}{3} + C = \frac{\tan^3 x}{3} + C$

Compute the following integrals using the guidelines for integrating powers of trigonometric functions. Use a CAS to check the solutions. (Note: Some of the problems may be done using techniques of integration learned previously.)

79. $\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx = \int \sin x \, dx - \int \cos^2 x \cdot \sin x \, dx$
 $u = \cos x$
 $du = -\sin x \, dx$
 $= -\cos x - \int u^2 \, du = -\cos x - \frac{u^3}{3} + C = -\cos x - \frac{\cos^3 x}{3} + C$

80. $\int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx = \int (1 - \sin^2 x) \cos x \, dx = \int \cos x \, dx - \int \sin^2 x \cos x \, dx$
 $u = \sin x$
 $du = \cos x \, dx$
 $= \sin x - \int u^2 \, du = \sin x - \frac{u^3}{3} + C = \sin x - \frac{\sin^3 x}{3} + C$

81. $\int \sin x \cos x \, dx$ $u = \sin x$
 $du = \cos x \, dx$
 $= \int u \, du = \frac{u^2}{2} + C = \frac{\sin^2 x}{2} + C$ (or $-\frac{\cos^2 x}{2} + C$)

82. $\int \cos^5 x \, dx = \int \cos^2 x \cos^3 x \cdot \cos x \, dx = \int (1 - \sin^2 x)(1 - \sin^2 x) \cos x \, dx$
 $u = \sin x$
 $du = \cos x \, dx$
 $= \int (1 - u^2)(1 - u^2) \, du = \int (1 - 2u^2 + u^4) \, du$
 $= u - \frac{2}{3} u^3 + \frac{u^5}{5} + C = \sin x - \frac{2}{3} \sin^3 x + \frac{\sin^5 x}{5} + C$

$$83. \int \sin^5 x \cos^2 x \, dx = \int \sin x \cdot \sin^2 x \cdot \sin^2 x \cos^2 x \, dx = \int \sin x (1 - \cos^2 x) (1 - \cos^2 x) \cos^2 x \, dx$$

$$\begin{aligned} & \boxed{u = \cos x} \\ & \boxed{du = -\sin x \, dx} \end{aligned} \rightarrow = \int - (1 - u^2) (1 - u^2) u^2 \, du = \int - (u^2 - 2u^4 + u^6) \, du$$

$$= -\frac{u^3}{3} + \frac{2}{5} u^5 - \frac{u^7}{7} + C = -\frac{\cos^3 x}{3} + \frac{2}{5} \cos^5 x - \frac{\cos^7 x}{7} + C$$

$$84. \int \sin^3 x \cos^3 x \, dx = \int \sin^2 x \cdot \cos^2 x \cos x \, dx = \int \sin^2 x (1 - \sin^2 x) \cdot \cos x \, dx$$

$$\begin{aligned} & u = \sin x \\ & du = \cos x \, dx \end{aligned} = \int u^2 (1 - u^2) \, du = \int u^2 - u^4 \, du = \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

$$85. \int \sqrt{\sin x} \cos x \, dx \quad u = \sin x; \, du = \cos x \, dx$$

$$= \int \sqrt{u} \, du = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} \sqrt{\sin^3 x} + C$$

$$91. \int \sec^4 x \, dx = \int \sec^2 x \sec^2 x \, dx = \int (1 + \tan^2 x) \sec^2 x \, dx$$

$$\boxed{u = \tan x} \quad \boxed{du = \sec^2 x \, dx} \rightarrow \int (1 + u^2) \, du = u + \frac{u^3}{3} + C = \tan x + \frac{\tan^3 x}{3} + C$$

Use the double-angle formulas to evaluate the following integrals.

$$97. \int_0^{\pi} \sin^2 x \, dx$$

$$= \int_0^{\pi} \frac{1 - \cos(2x)}{2} \, dx$$

$$= \int_0^{\pi} \frac{1}{2} - \frac{\cos(2x)}{2} \, dx$$

$$= \left[\frac{x}{2} - \frac{\sin(2x)}{4} \right]_0^{\pi}$$

$$= \left[\frac{\pi}{2} - \frac{\sin(2\pi)}{4} \right] - \left[\frac{0}{2} - \frac{\sin(0)}{4} \right]$$

$$= \left[\frac{\pi}{2} - 0 \right] - [0 - 0] = \frac{\pi}{2}$$

$$98. \int_0^{\pi} \sin^4 x \, dx$$

$$= \int_0^{\pi} \left(\frac{1 - \cos(2x)}{2} \right) \left(\frac{1 - \cos(2x)}{2} \right) \, dx$$

$$= \int_0^{\pi} \left(\frac{1}{4} - \frac{\cos(2x)}{2} + \frac{\cos^2(2x)}{4} \right) \, dx$$

$$= \int_0^{\pi} \frac{1}{4} \, dx - \int_0^{\pi} \frac{\cos(2x)}{2} \, dx + \int_0^{\pi} \frac{\cos^2(2x)}{4} \, dx$$

$$= \int_0^{\pi} \frac{1}{4} \, dx - \int_0^{\pi} \frac{\cos(2x)}{2} \, dx + \int_0^{\pi} \frac{1}{4} \left(\frac{1 + \cos(4x)}{2} \right) \, dx$$

$$= \int_0^{\pi} \frac{1}{4} \, dx - \int_0^{\pi} \frac{\cos(2x)}{2} \, dx + \int_0^{\pi} \frac{1}{8} \, dx + \int_0^{\pi} \frac{\cos(4x)}{8} \, dx$$

$$= \int_0^{\pi} \frac{3}{8} \, dx - \left[\frac{\sin(2x)}{4} \right]_0^{\pi} + \left[\frac{\sin(4x)}{32} \right]_0^{\pi}$$

$$= \left[\frac{3}{8} x \right]_0^{\pi} - \left[\frac{\sin(2\pi)}{4} - \frac{\sin(0)}{4} \right] + \left[\frac{\sin(4\pi)}{32} - \frac{\sin(0)}{32} \right] = \frac{3}{8} \pi$$

$$100. \int \sin^2 x \cos^2 x \, dx = \int \left(\frac{1 - \cos(2x)}{2} \right) \left(\frac{1 + \cos(2x)}{2} \right) dx$$

$$= \int \frac{1 - \cos^2(2x)}{4} dx = \int \frac{\sin^2(2x)}{4} dx = \int \frac{1}{4} \left(\frac{1 - \cos(4x)}{2} \right) dx$$

$$= \int \left(\frac{1}{8} - \frac{\cos(4x)}{8} \right) dx = \frac{x}{8} - \frac{\sin(4x)}{32} + C$$