

Mat 1575 HW6 (§3.1)

Find the integral by using the simplest method. Not all problems require integration by parts.

$$7. \int \ln x \, dx = x \ln x - \int x \frac{dx}{x}$$

$$u = \ln x \begin{cases} dv = dx \\ du = \frac{dx}{x} \end{cases} \quad v = x$$

$$= x \ln x - \int dx = x \ln x - x + C$$

$$8. \int x \cos x \, dx = x \sin(x) - \int \sin(x) \, dx$$

$$u = x \begin{cases} dv = \cos x \, dx \\ du = dx \end{cases} \quad v = \sin(x)$$

$$= x \sin(x) + \cos(x) + C$$

$$13. \int x e^{-x} \, dx = -x e^{-x} - \int -e^{-x} \, dx$$

$$u = x \begin{cases} dv = e^{-x} \, dx \\ du = dx \end{cases} \quad v = -e^{-x}$$

$$= -x e^{-x} + \int e^{-x} \, dx = -x e^{-x} - e^{-x} + C$$

$$15. \int x^2 \cos x \, dx = x^2 \sin(x) - \int 2x \sin(x) \, dx = x^2 \sin(x) - \left[-2x \cos(x) - \int 2 \cos(x) \, dx \right]$$

$$u = x^2 \begin{cases} dv = \cos x \, dx \\ du = 2x \, dx \end{cases} \quad v = \sin(x)$$

$$u = 2x \begin{cases} dv = \sin(x) \, dx \\ du = 2 \, dx \end{cases} \quad v = -\cos(x)$$

$$= x^2 \sin(x) + 2x \cos(x) + \int 2 \cos(x) \, dx$$

$$= x^2 \sin(x) + 2x \cos(x) + 2 \sin(x) + C$$

$$16. \int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{dx}{x} = \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$$

$$u = \ln x \begin{cases} dv = x \, dx \\ du = \frac{dx}{x} \end{cases} \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C$$

$$19. \int e^x \sin x \, dx = e^x \sin(x) - \int e^x \cos(x) \, dx = e^x \sin(x) - \left[e^x \cos(x) - \int -e^x \sin(x) \, dx \right]$$

$$u = \sin(x) \begin{cases} dv = e^x \, dx \\ du = \cos(x) \, dx \end{cases} \quad v = e^x$$

$$u = \cos(x) \begin{cases} dv = e^x \, dx \\ du = -\sin(x) \, dx \end{cases} \quad v = e^x$$

$$= e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) \, dx$$

$$\Rightarrow \int e^x \sin(x) \, dx = e^x \sin(x) - e^x \cos(x) - \int e^x \sin(x) \, dx$$

$$\Rightarrow 2 \int e^x \sin(x) \, dx = e^x \sin(x) - e^x \cos(x) + C \Rightarrow \int e^x \sin(x) \, dx = \frac{1}{2} [e^x \sin(x) - e^x \cos(x)] + C$$

$$20. \int e^x \cos x \, dx = \frac{1}{2} [e^x \sin(x) + e^x \cos(x)] + C \quad (\text{check classwork or, item 9})$$

$$27. \int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \frac{dx}{x} = \frac{x^3}{3} \ln x - \int \frac{x^2}{3} dx$$

$$= \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$$

$$u = \ln x \quad dv = x^2 dx$$

$$du = \frac{dx}{x} \quad v = \frac{x^3}{3}$$

$$31. \int x^2 \sin x \, dx = -x^2 \cos(x) - \int -2x \cos(x) dx = -x^2 \cos(x) + \int 2x \cos(x) dx$$

$$u = x^2 \quad dv = \sin(x) dx$$

$$du = 2x dx \quad v = -\cos(x)$$

$$u = 2x \quad dv = \cos(x) dx$$

$$du = 2 dx \quad v = \sin(x)$$

$$= -x^2 \cos(x) + 2x \sin(x) - \int 2 \sin(x) dx$$

$$= -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C$$

Compute the definite integrals. Use a graphing utility to confirm your answers.

$$38. \int_{1/e}^1 \ln x \, dx = \left[x \ln x \right]_{1/e}^1 - \int_{1/e}^1 x \frac{dx}{x} = \left[1 \ln(1) - \frac{1}{e} \ln\left(\frac{1}{e}\right) \right] - x \Big|_{1/e}^1$$

$$u = \ln x \quad dv = dx$$

$$du = \frac{dx}{x} \quad v = x$$

$$= \left[0 - \frac{1}{e} (1) \right] - \left[1 - \frac{1}{e} \right] = \frac{1}{e} + \frac{1}{e} = \frac{2}{e} - 1$$

$$\ln(1) = 0$$

$$\ln\left(\frac{1}{e}\right) = \ln(e^{-1})$$

$$= -\ln(e) = -1$$

$$42. \int_0^\pi x \cos x \, dx = \left[x \sin(x) \right]_0^\pi - \int_0^\pi \sin(x) dx = \left[x \sin(x) \right]_0^\pi + \left[\cos(x) \right]_0^\pi$$

$$u = x \quad dv = \cos(x) dx$$

$$du = dx \quad v = \sin(x)$$

$$= \left[\pi \sin(\pi) - 0 \cdot \sin(0) \right] + \left[\cos(\pi) - \cos(0) \right]$$

$$= \left[0 - 0 \right] + \left[-1 - (1) \right] = -2$$

$$\sin(\pi) = 0, \sin(0) = 0$$

$$\cos(\pi) = -1, \cos(0) = 1$$

$$43. \int_{-\pi}^{\pi} x \sin x \, dx = \left[-x \cos(x) \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -\cos(x) \, dx$$

$$= \left[-x \cos(x) \right]_{-\pi}^{\pi} + \left[\sin(x) \right]_{-\pi}^{\pi}$$

$$= \left[-\pi \cos(\pi) + (-\pi) \cos(-\pi) \right] + \left[\sin(\pi) - \sin(-\pi) \right]$$

$$\cos(\pi) = -1, \cos(-\pi) = -1$$

$$\sin(\pi) = 0, \sin(-\pi) = 0$$

$$= \left[-\pi(-1) + (-\pi)(-1) \right] + [0 - 0] = 2\pi$$

$$45. \int_0^{\pi/2} x^2 \sin x \, dx = \left[-x^2 \cos(x) \right]_0^{\pi/2} - \int_0^{\pi/2} -2x \cos(x) \, dx = \left[x^2 \cos(x) \right]_0^{\pi/2} + \int_0^{\pi/2} 2x \cos(x) \, dx$$

$$u = x^2 \quad dv = \sin(x) \, dx \\ du = 2x \, dx \quad v = -\cos(x)$$

$$u = 2x \quad dv = \cos(x) \, dx \\ du = 2 \, dx \quad v = \sin(x)$$

$$= \left[-x^2 \cos(x) \right]_0^{\pi/2} + \left[2x \sin(x) \right]_0^{\pi/2} - \int_0^{\pi/2} 2 \sin(x) \, dx$$

$$= \left[-x^2 \cos(x) \right]_0^{\pi/2} + \left[2x \sin(x) \right]_0^{\pi/2} + \left[2 \cos(x) \right]_0^{\pi/2}$$

$$= \left[-\frac{\pi^2}{4} \cos\left(\frac{\pi}{2}\right) + 0 \cos(0) \right] + \left[\pi \sin\left(\frac{\pi}{2}\right) - 2 \cdot 0 \cdot \sin(0) \right] + \left[2 \cos\left(\frac{\pi}{2}\right) - 2 \cos(0) \right]$$

$$\cos\left(\frac{\pi}{2}\right) = 0 \\ \sin\left(\frac{\pi}{2}\right) = 1$$

$$= [0 + 0] + [\pi - 0] + [0 - 2] = \pi - 2$$