

Mat 1575 HW 5 (§1.6)

In the following exercises, compute each indefinite integral.

NOTE: $2^x = e^{\ln 2^x} = e^{x \ln 2}$

$$\begin{aligned} 320. \int e^{2x} dx & \quad u=2x, du=2dx \\ & \quad \frac{du}{2} = dx \\ & = \int e^u \frac{du}{2} \\ & = \frac{e^u}{2} + C \\ & = \frac{e^{2x}}{2} + C \end{aligned}$$

$$\begin{aligned} 321. \int e^{-3x} dx & \quad u=-3x, du=-3dx \\ & \quad \frac{du}{-3} = dx \\ & = \int e^u \frac{du}{-3} \\ & = \frac{e^u}{-3} + C \\ & = \frac{e^{-3x}}{-3} + C \end{aligned}$$

$$\begin{aligned} 322. \int 2^x dx & = \int e^{\ln 2^x} dx \\ & = \int e^{x \ln 2} dx \\ & = \frac{e^{x \ln 2}}{\ln 2} + C \\ & = \frac{2^x}{\ln 2} + C \end{aligned}$$

$$\begin{aligned} 325. \int \frac{2}{x} dx & \\ & = 2 \ln|x| + C \end{aligned}$$

$$\begin{aligned} 327. \int \frac{1}{\sqrt{x}} dx & \\ & = 2x^{\frac{1}{2}} + C \end{aligned}$$

In the following exercises, find each indefinite integral by using appropriate substitutions.

$$\begin{aligned} 328. \int \frac{\ln x}{x} dx & = \int u du = \frac{u^2}{2} + C = \frac{(\ln x)^2}{2} + C \\ u & = \ln x, du = \frac{dx}{x} \end{aligned}$$

$$\begin{aligned} 330. \int \frac{dx}{x \ln x} \quad (x > 1) & \Rightarrow \int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln|u| + C = \ln|\ln x| + C \\ u & = \ln x, du = \frac{dx}{x} \end{aligned}$$

$$\begin{aligned} 332. \int \tan \theta d\theta & = \int \frac{\sin \theta}{\cos \theta} d\theta = \int \frac{-du}{u} = -\ln|u| + C = -\ln|\cos \theta| + C \\ u & = \cos \theta, du = -\sin \theta d\theta, -du = \sin \theta d\theta \end{aligned}$$

$$\begin{aligned} 335. \int \ln(\cos x) \tan x dx & = \int u \cdot (-du) = \int -u du = -\frac{u^2}{2} + C \\ u & = \ln(\cos x), du = \frac{-\sin x}{\cos x} dx = -\tan x dx \\ & \Rightarrow -du = \tan x dx \\ & = -\frac{(\ln|\cos x|)^2}{2} + C \end{aligned}$$

$$337. \int x^2 e^{-x^3} dx = \int e^u \frac{du}{-3} = \frac{e^u}{-3} + C = \frac{e^{-x^3}}{-3} + C$$

$$u = -x^3, du = -3x^2 dx, \frac{du}{-3} = x^2 dx$$

$$338. \int e^{\sin x} \cos x dx = \int e^u du = e^u + C = e^{\sin(x)} + C$$

$$u = \sin(x), du = \cos(x) dx$$

In the following exercises, evaluate the definite integral.

$$355. \int_1^2 \frac{1+2x+x^2}{3x+3x^2+x^3} dx$$

$$\begin{aligned} u &= 3x+3x^2+x^3 \\ du &= 3+6x+3x^2 dx \\ \frac{du}{3} &= 1+2x+x^2 dx \\ &= \int_{u(1)}^{u(2)} \frac{1}{u} \frac{du}{3} \\ &= \int_7^{26} \frac{du}{3u} \\ &= \frac{\ln|u|}{3} \Big|_7^{26} \\ &= \frac{1}{3} (\ln 26 - \ln 7) \end{aligned}$$

$$356. \int_0^{\pi/4} \tan x dx$$

$$\begin{aligned} u &= \cos(x) \\ du &= -\sin(x) dx \\ &= \int_0^{\pi/4} \frac{\sin(x)}{\cos(x)} dx = \int_1^{\frac{\sqrt{2}}{2}} \frac{-du}{u} \\ &= \int_{\frac{\sqrt{2}}{2}}^1 \frac{du}{u} = \ln|u| \Big|_{\frac{\sqrt{2}}{2}}^1 \\ &= \ln 1 - \ln \frac{\sqrt{2}}{2} \\ &= -\ln \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} u &= \sin(x) + \cos(x) \\ du &= (\cos(x) - \sin(x)) dx \\ -du &= (-\cos(x) + \sin(x)) dx \\ 357. \int_0^{\pi/3} \frac{\sin x - \cos x}{\sin x + \cos x} dx \\ &= \int_1^{\frac{1+\sqrt{3}}{2}} \frac{-du}{u} \\ &= \int_{\frac{1+\sqrt{3}}{2}}^1 \frac{du}{u} \\ &= \ln|u| \Big|_{\frac{1+\sqrt{3}}{2}}^1 \\ &= \ln 1 - \ln\left(\frac{1+\sqrt{3}}{2}\right) = -\ln\left(\frac{1+\sqrt{3}}{2}\right) \end{aligned}$$

$$358. \int_{\pi/6}^{\pi/2} \csc x dx$$

$$\begin{aligned} &= \left[\ln |\csc(x) - \cot(x)| \right]_{\pi/6}^{\pi/2} \\ &= \ln \left| \csc\left(\frac{\pi}{2}\right) - \cot\left(\frac{\pi}{2}\right) \right| - \ln \left| \csc\left(\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{6}\right) \right| \\ &= \ln \left| \frac{1}{\sin(\frac{\pi}{2})} - \frac{\cos(\frac{\pi}{2})}{\sin(\frac{\pi}{2})} \right| - \ln \left| \frac{1}{\sin(\frac{\pi}{6})} - \frac{\cos(\frac{\pi}{6})}{\sin(\frac{\pi}{6})} \right| \\ &= \ln |1-0| - \ln |2-\sqrt{3}| \\ &= -\ln |2-\sqrt{3}| \end{aligned}$$

$$359. \int_{\pi/4}^{\pi/3} \cot x dx$$

$$\begin{aligned} u &= \sin(x), du = \cos(x) dx \\ &= \int_{\pi/4}^{\pi/3} \frac{\cos(x)}{\sin(x)} dx = \int_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} \frac{du}{u} \\ &= \left[\ln|u| \right]_{\frac{\sqrt{2}}{2}}^{\frac{\sqrt{3}}{2}} = \ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{\sqrt{2}}{2}\right) \end{aligned}$$

In the following exercises, integrate using the indicated substitution.

360. $\int \frac{x}{x-100} dx; u = x - 100$

$du = dx, \quad x = u + 100$

$= \int \frac{u+100}{u} du = \int \frac{u}{u} + \frac{100}{u} du$

$= \int \left(1 + \frac{100}{u}\right) du$

$= u + 100 \ln|u| + C$

361. $\int \frac{y-1}{y+1} dy; u = y + 1$

$du = dy, \quad y-1 = (u+1)-2 = u-2$

$\int \frac{u-2}{u} du$

$= \int \frac{u}{u} - \frac{2}{u} du = \int \left(1 - \frac{2}{u}\right) du$

$= u - 2 \ln|u| + C$

362. $\int \frac{1-x^2}{3x-x^3} dx; u = 3x - x^3$

$du = 3 - 3x^2 dx; \frac{du}{3} = 1 - x^2 dx$

$\int \frac{1}{3} \frac{du}{u} = \frac{1}{3} \ln|u| + C$

$= \frac{1}{3} \ln|3x - x^3| + C$

363. $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx; u = \sin x - \cos x \quad du = (\cos x + \sin x) dx$

$= \int \frac{du}{u} = \ln|u| + C = \ln|\sin x - \cos x| + C$