# MAT1575, Classwork4, Fall2026

## *1.3 The Fundamental Theorem of Calculus (p.40-56)*

1. *Definition*: Average value of the function.

Let $f(x)$ be continuous on the interval $[a, b]$. Then, the average value of the function $f(x)$, denoted $f_{ave}$, on $[a, b]$ is given by

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$$

2. *Example*: Find the average value of $f(x) = 6 - 2x$ on the interval $[0, 3]$.

$$f_{ave} = \frac{1}{3-0}\int_0^3 (6-2x)dx = \frac{1}{3}\int_0^3 (6-2x)dx = ?$$

3. *Theorem*: **The Mean Value Theorem for Integrals.**

If $f(x)$ is continuous on interval $[a, b]$, then there is at least one point $c \in [a, b]$ such that

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx \qquad or \qquad \int_a^b f(x)dx = f(c)(b-a)$$

4. *Example*. Find the average value of function $f(x) = \frac{x}{2}$ over the interval $[0, 6]$ and find a $c \in [0,6]$ equals the average value of the function over $[0, 6]$.

5. *Theorem*. **Fundamental Theorem of Calculus**, Part 1.

If $f(x)$ is continuous on interval $[a, b]$, and the function $F(x)$ is defined by

$$F(x) = \int_a^x f(t)dt \text{, then } F'(x) = f(x) \text{ over } [a, b].$$

*Proof*: we have a ________________ such that

$F'(x) =$

6. *Example*. Use the Fundamental Theorem of Calculus, Part 1 to find the derivative of

$(a)\ G(x) = \int_1^x \frac{1}{t^3+1}dt$ $\qquad (b)\ F(r) = \int_0^r \sqrt{x^2+4}\,dx$ $\qquad (c) H(x) = \int_1^{\sqrt{x}} \sin(t)\,dt$ $\qquad (d) K(x) = \int_x^{2x} t^3\,dt$

*Observation*: To make **Fundamental Theorem of Calculus, Part 1** work, we need to have

(a) ________________________________

(b) ________________________________

________________________________

(c) ________________________________

7. *Theorem*: **Fundamental Theorem of Calculus, Part 2: The Evaluation Theorem**

If $f(x)$ is continuous on interval $[a, b]$, and the function $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = \left. F(x)\right|_a^b = F(b) - F(a). \quad \text{or } \left[F(x)\right]_a^b = F(b) - F(a)$$

*Proof*: Let $P = \{x_i\}$ for $i = 1,2, \ldots, n$ be a partition on $[a, b]$. By Mean Value Thm, we can find a $c_i \in [x_{i-1}, x_i]$

s.t. $\dfrac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}} = F'(c_i) \Rightarrow F(x_i) - F(x_{i-1}) = F'(c_i)(x_i - x_{i-1}) = f(c_i)(x_i - x_{i-1})$

$= f(c_i) \cdot \Delta x$

Then $F(b) - F(a) = \sum_{i=1}^{n} f(c_i)\Delta x \Rightarrow \lim_{n\to\infty}(F(b) - F(a)) = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x$

Take limit on both sides and get $F(b) - F(a) = \int_a^b f(x)\,dx$

*observation:*

*The drawback of this method*:

8. Evaluate $\int_{-2}^{2}(t^2 - 4)dt$

$$= \left[\frac{t^{2+1}}{3} - 4t\right]_{-2}^{2}$$

$t=2$, $t=-2$

$$= \left[\frac{(2)^3}{3} - 4\cdot 2\right] - \left[\frac{(-2)^3}{3} - 4\cdot(-2)\right]$$

$$= \left[\frac{8}{3} - 8\right] - \left[\frac{-8}{3} + 8\right] = \frac{8}{3} - 8 + \frac{8}{3} - 8 = \frac{16}{3} - 16 = -\frac{32}{3}$$

9. Evaluate $\int_1^9 \frac{y-1}{\sqrt{y}}dy$

$$= \int_1^9 \left(\frac{y}{\sqrt{y}} - \frac{1}{\sqrt{y}}\right)dy = \int_1^9 \left(y^{\frac{1}{2}} - y^{-\frac{1}{2}}\right)dy = \left[\frac{2}{3}y^{\frac{3}{2}} - 2y^{\frac{1}{2}}\right]_1^9$$

$y^{\frac{3}{2}} = (\sqrt{y})^3$

$y^{\frac{1}{2}} = \sqrt{y}$

$y=9$, $y=1$

$$= \left[\frac{2}{3}(\sqrt{9})^3 - 2\cdot\sqrt{9}\right] - \left[\frac{2}{3}(\sqrt{1})^3 - 2\cdot\sqrt{1}\right]$$

$$= \left[\frac{2}{3}27 - 2\cdot 3\right] - \left[\frac{2}{3} - 2\right]$$

$$= 18 - 6 - \frac{2}{3} + 2 = 14 - \frac{2}{3} = \frac{40}{3}$$

10. James skates along a long, straight track at a velocity of $f(t) = 10 + \cos\left(\frac{\pi}{2}t\right)$ ft/sec. Find how far James has gone after 5 seconds.