Test2 Review, MAT 2440 Professor Chiu

- This review consists of 7 set of questions.
- You have 60 minutes to complete this review.
- Show all work and justify your answers.
- Wishing you success.

1. Given the pseudocode of the insertion sort algorithm:

procedure *insertionsort*($a_1, a_2, \cdots, a_n$: real numbers with $n \geq 2$)
$n :=$ the length of $\{a_i\}$
  for $i := 2$ **to** n
    $j := 1$
    while ($a_i > a_j$ and $i > j$)
      $j := j + 1$
    $m := a_i$
    for $k := 0$ **to** $i - j - 1$
      $a_{i-k} := a_{i-k-1}$
    $a_j := m$
$\{a_1, a_2, \cdots, a_n$ is in increasing order$\}$

Use the insertion sort to sort $3, 1, 5, 7, 4$ in increasing order, showing the lists obtained at each step with all the details.

2. Given the pseudocode of the binary search algorithm:

procedure *binary_search*(x: integer, $a_1, a_2, \cdots, a_n$: distinct integers)
$n :=$ the length of $\{a_i\}$; $i := 1$ (left end location); $j := n$ (right end location)
  while ($i \leq j$)
    m:= $\left\lfloor \frac{i+j}{2} \right\rfloor$
    if $x > a_m$ **then** $i := m + 1$
      else $j := m$
  if $x = a_i$ **then** *location* $:= i$
    else *location* $:= 0$
return *location* {either the subscript of the term that equals x, or 0 if x is not found.}

Use the binary search to search for 9 in the sequence $1, 3, 4, 5, 6, 8, 9, 11$, showing the lists obtained at each step with all the details.

3. Write the pseudocode for an algorithm that takes a list of n integers and produces as output the sum of numbers in the list.

4. Write the pseudocode for an algorithm that finds both the largest and smallest integers in a finite sequence of integers.

5. Find these five terms a_1, a_2, a_3, a_4, a_5 of each sequence.

(a) $a_0 = 1, a_n = 5 + \left\lfloor \frac{4a_{n-1}}{3} \right\rfloor$.

(b) $a_n = n^2 + \frac{5}{n+1}$.

(c) $a_1 = 1, a_2 = 3, a_{n+2} = a_{n+1} + a_n$.

(d) $a_0 = 2, a_n = 2n + a_{n-1}$.

6. Find the values of each of the sums.

$$(a) \sum_{i=1}^{10} \frac{1}{k(k+1)}$$

$$(b) \sum_{j=0}^{8} 2^{j+1} - 2^j$$

$$(c) \sum_{i=0}^{2} \sum_{j=1}^{3} i^2 + j$$

7. Determine whether each of these functions is bijection from $\mathbb{R}$ to $\mathbb{R}$. If yes, find its inverse function.

(a) $f(x) = \sqrt{2x+7}$

(b) $f(x) = 2x + 1$

(c) $f(x) = \frac{3x+2}{2x+1}$

(d) $f(x) = x^3$