

Test1 Review, MAT 2440 Professor Chiu

- This review consists of 8 set of questions.
- You have 60 minutes to complete this review.
- Show all work and justify your answers.
- Wishing you success.

1. Determine whether $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology.

2. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

3. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements ‘ x is an engineer,’ ‘ x is smart,’ and ‘ x is vain,’ respectively, where the domain consists of all people. Translate each of these statement into English.

(a) $\forall x \neg(P(x) \wedge Q(x))$. (b) $\exists x(R(x) \wedge \neg Q(x)) \rightarrow \exists y P(y)$.

4. (Follow-up to previous problem.) Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$.

(a) All smart people are vain. (b) No engineers are smart.

(c) There is a person that is both vain and an engineer.

5. Negate the following statements so that the negation appears only within the predicates.

(a) $\forall x \exists y P(x, y)$. (b) $\exists y(Q(y) \wedge \forall x \neg R(x, y))$

6. Determine whether the following arguments are valid. If the statement is correct, what rule of inference is being used? If it is not, what logical error occurs?

(a) All dogs are mammals. Spike is a dog. Therefore, Spike is a mammal.

(b) If it snows today, the university will be closed. The university is not closed today. Therefore, it did not show toady.

(c) If a band is a rock band, Quinn likes this band. Quinn likes the Punch Brothers. Therefore, the Punch Brothers is a rock band.

(d) Consider the argument form:

$$\begin{array}{l} p \wedge q \\ p \rightarrow r \\ q \rightarrow s \\ \hline \therefore r \wedge s \end{array}$$

7. Show that, for every integer n , n^2 is even if and only if n is even.

8. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

(a) a proof by contraposition. (b) a proof by contradiction.