MAT2440 Project Spring2025

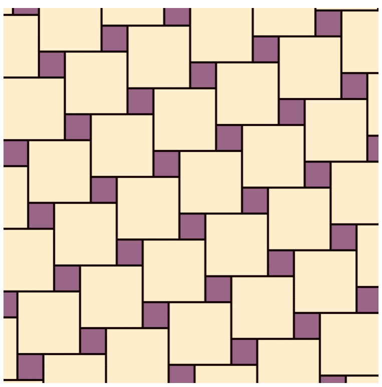
1. In section 5.1, we introduce the Mathematical induction which could be used to prove statements "$P(n)$ is true" for all positive integers where $P(n)$ is a propositional function.

   Use the induction to prove the following statement:

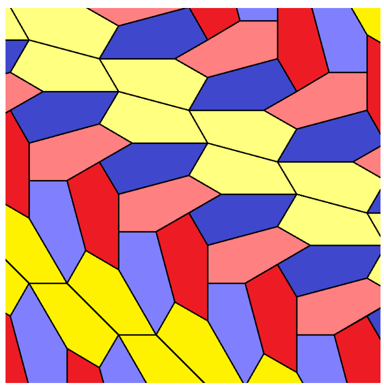
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ for all } n \in \mathbb{Z}^+$$

2. Introduction of Tessellation.

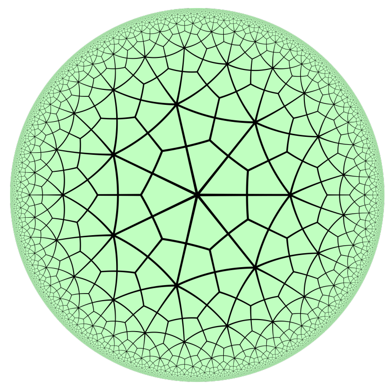
   A tessellation or tiling is the covering of a surface using one or more geometric shapes with no overlaps and no gaps.



(a) A Pythagorean tiling



(b) A convex monohedral pentagonal tiling



(c) A deltoidal triheptagonal tiling

Figure 1: Examples of tessellation

From 1., we know

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2 \text{ for all } n \in \mathbb{Z}^+.$$

If we rewrite this statement as the following:

$$1 \times 1^2 + 2 \times 2^2 + 3 \times 3^2 + \cdots + n \times n^2 = 1 \times \left(\frac{n(n+1)}{2}\right)^2,$$

and see the square numbers as the areas of squares ($1^2, 2^2, 3^2, \cdots, n^2$, and $\left(\frac{n(n+1)}{2}\right)^2$ are the area of squares with length $1, 2, 3, \cdots, n$, and $\frac{n(n+1)}{2}$, respectively).

Theoretically speaking, for a given integer number n, there is a tessellation of a $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$ square by using an 1×1 square, two of 2×2 squares, three of 3×3 squares, $\cdots$, and n of $n \times n$ squares to cover it.

For example, a possible tessellation for $n = 8$ is in Figure 2. This is a 36×36 square covered by an 1×1 square, two of 2×2 squares, three of 3×3 squares, four of 4×4 squares, five of 5×5 squares, six of 6×6 squares, seven of 7×7 squares, and eight of 8×8 squares.

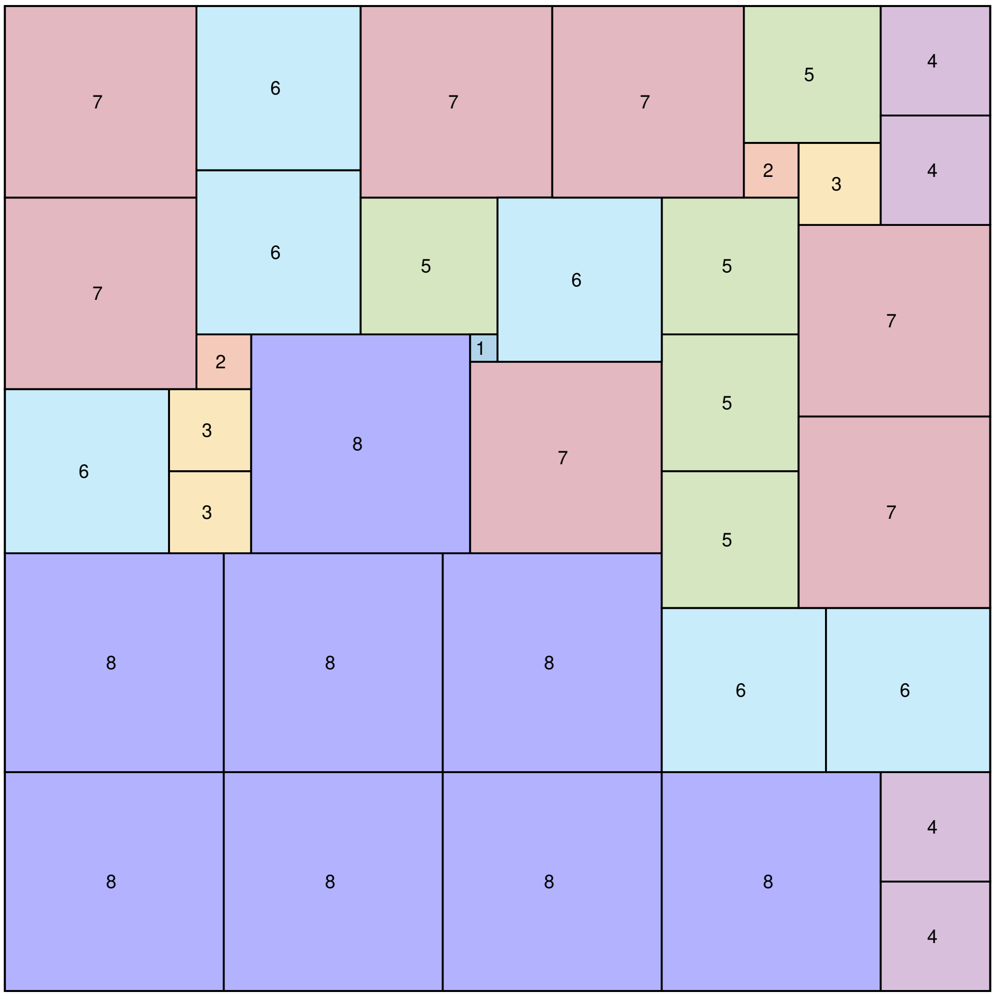


Figure 2: A square tessellation of a 36×36 square.

Please provide a pseudocode of the algorithm to find this kind of tessellation.

3. Write a code based on your algorithm to find this kind of tessellations. Please provide a tessellation for $n = 8$ which is differenet from the one in Figure 2.