Section 2.2

3. a) $\{0,1,2,3,4,5,6\}$ **b)** $\{3\}$ **c)** $\{1,2,4,5\}$ **d)** $\{0,6\}$

15. **b)**

A	B	$A \cup B$	$\overline{A \cup B}$	$\overline{A}$	$\overline{B}$	$\overline{A} \cap \overline{B}$
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

19. **b)**

A	B	C	$A \cap B \cap C$	$\overline{A \cap B \cap C}$	$\overline{A}$	$\overline{B}$	$\overline{C}$	$\overline{A} \cup \overline{B} \cup \overline{C}$
1	1	1	1	0	0	0	0	0
1	1	0	0	1	0	0	1	1
1	0	1	0	1	0	1	0	1
1	0	0	0	1	0	1	1	1
0	1	1	0	1	1	0	0	1
0	1	0	0	1	1	0	1	1
0	0	1	0	1	1	1	0	1
0	0	0	0	1	1	1	1	1

23. Prove the first associative law from Table 1 by showing that if A, B, and C are sets, then $A \cup (B \cup C) = (A \cup B) \cup C$. To show $A \cup (B \cup C) = (A \cup B) \cup C$, we need to prove
(a) $A \cup (B \cup C) \subseteq (A \cup B) \cup C$ and (b) $A \cup (B \cup C) \supseteq (A \cup B) \cup C$

Proof (a): Let $x \in A \cup (B \cup C)$
$\Rightarrow x \in A$ or $x \in B \cup C$
$\Rightarrow x \in A$ or $x \in B$ or $x \in C$
$\Rightarrow x \in A \cup B$ or $x \in C$
$\Rightarrow x \in (A \cup B) \cup C$
$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C$

Proof (b): Let $x \in (A \cup B) \cup C$
$\Rightarrow x \in A \cup B$ or $x \in C$
$\Rightarrow x \in A$ or $x \in B$ or $x \in C$
$\Rightarrow x \in A$ or $x \in B \cup C$
$\Rightarrow x \in A \cup (B \cup C)$
$\therefore A \cup (B \cup C) \supseteq (A \cup B) \cup C$

By (a)(b), we have $A \cup (B \cup C) = (A \cup B) \cup C$.

24. Prove the second associative law from Table 1 by showing that if A, B, and C are sets, then $A \cap (B \cap C) = (A \cap B) \cap C$.

To show $A \cap (B \cap C) = (A \cap B) \cap C$, we need to prove

(a) $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ and (b) $A \cap (B \cap C) \supseteq (A \cap B) \cap C$

proof (a): Let $x \in A \cap (B \cap C)$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow x \in A \cap B$ and $x \in C$

$\Rightarrow x \in (A \cap B) \cap C$

Therefore, $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

proof (b): Let $x \in (A \cap B) \cap C$

$\Rightarrow x \in A \cap B$ and $x \in C$

$\Rightarrow x \in A$ and $x \in B$ and $x \in C$

$\Rightarrow x \in A$ and $x \in B \cap C$

$\Rightarrow x \in A \cap (B \cap C)$

Therefore, $(A \cup B) \cup C \subseteq A \cap (B \cap C)$

By (a) & (b), we prove that $A \cap (B \cap C) = (A \cap B) \cap C$

25. Prove the first distributive law from Table 1 by showing that if A, B, and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

To show $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, we need to prove

(a) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ and (b) $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

proof (a): Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in B \cap C$

$\Rightarrow x \in A$ or $x \in B$ and $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

Therefore, $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

proof (b): Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow x \in A \cup (B \cap C)$

Therefore, $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

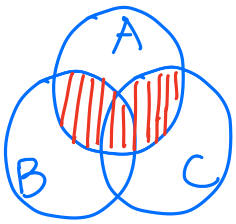
In conclusion, by (a) & (b), $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

28. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) $A \cap (B \cup C)$ **b)** $\overline{A} \cap \overline{B} \cap \overline{C}$

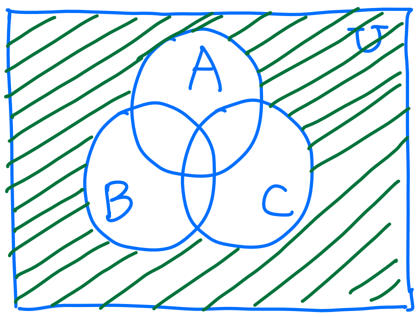
c) $(A - B) \cup (A - C) \cup (B - C)$

a)



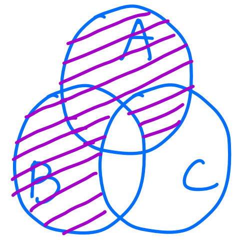
$A \cap (B \cup C)$

b)



$\overline{A} \cap \overline{B} \cap \overline{C}$

c)



$(A-B) \cup (A-C) \cup (B-C)$

***52.** Show that if A, B, and C are sets, then

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

Sol. Assume, for two sets S, T, that this is true:

$$|S \cup T| = |S| + |T| - |S \cap T|.$$

Now, without loss of generality, let $S = A$ and $T = B \cup C$, we have

$$|A \cup (B \cup C)| = |A| + |B \cup C| - |A \cap (B \cup C)|$$

Union of two sets

$$= |A| + |B \cup C| - |(A \cap B) \cup (A \cap C)|$$

$$= |A| + |B \cup C| - \left(|A \cap B| + |A \cap C| - |A \cap B \cap A \cap C|\right)$$

$$= |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C| \quad \#$$

To show $|S \cup T| = |S| + |T| - |S \cap T|$, we have

$|S| = |S \cap T| + |S - T|$ and $|T| = |S \cap T| + |T - S|$

$$\Rightarrow |S| + |T| = \underbrace{|S \cap T| + |S - T| + |T - S|} + |S \cap T|$$

$$\Rightarrow |S| + |T| = |S \cup T| + |S \cap T|$$

$$\Rightarrow |S| + |T| - |S \cap T| = |S \cup T| \quad \#$$

53. Let $A_i = \{1, 2, 3, \ldots, i\}$ for $i = 1, 2, 3, \ldots$. Find

a) $\bigcup_{i=1}^{n} A_i$.

b) $\bigcap_{i=1}^{n} A_i$.

Sol: $A_1 = \{1\}$

$A_2 = \{1, 2\}$

$A_3 = \{1, 2, 3\}$

$\vdots$

$A_n = \{1, 2, 3, 4, \cdots, n\}$

a) $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$

$= \{1, 2, 3, 4, \cdots, n\}$

including all elements

b) $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$

$= \{1\}$.

only has the common element(s)

58. Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Express each of these sets with bit strings where the ith bit in the string is 1 if i is in the set and 0 otherwise.

bit string

a) $\{3, 4, 5\}$ $\Rightarrow$ 0 0 1 1 1 0 0 0 0 0

b) $\{1, 3, 6, 10\}$ $\Rightarrow$ 1 0 1 0 0 1 0 0 0 1

c) $\{2, 3, 4, 7, 8, 9\}$ $\Rightarrow$ 0 1 1 1 0 0 1 1 1 0

59. Using the same universal set as in the last exercise, find the set specified by each of these bit strings.

a) 11 1100 1111 $\Rightarrow \{1, 2, 3, 4, 7, 8, 9, 10\}$

b) 01 0111 1000 $\Rightarrow \{2, 4, 5, 6, 7\}$

c) 10 0000 0001 $\Rightarrow \{1, 10\}$

60. What subsets of a finite universal set do these bit strings represent?

a) the string with all zeros $\Rightarrow$ empty set.

b) the string with all ones $\Rightarrow$ universal set.