Section 2.1

1. **a)** $\{-1, 1\}$ **b)** $\{1,2,3,4,5,6,7,8,9,10,11\}$ **c)** $\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$ **d)** $\emptyset$

7. Determine whether each of these pairs of sets are equal.

a) $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}, \{5, 3, 1\}$

b) $\{\{1\}\}, \{1, \{1\}\}$ **c)** $\emptyset, \{\emptyset\}$

Sol: a) Yes b) NO, since 1 and $\{1\}$ are different.

c) NO. since ϕ and $\{\phi\}$ are different

13. Determine whether each of these statements is true or false.

a) $x \in \{x\}$ **b)** $\{x\} \subseteq \{x\}$ **c)** $\{x\} \in \{x\}$

d) $\{x\} \in \{\{x\}\}$ **e)** $\emptyset \subseteq \{x\}$ **f)** $\emptyset \in \{x\}$

Sol:

a) True, "the element" x belongs to the set $\{x\}$

b) True, a set is its own subset.

c) false, "the element" $\{x\}$ is not in the set $\{x\}$

d) True, "the element" $\{x\}$ belongs to the set $\{\{x\}\}$.

e) True, empty set is subset of any set.

f) false, "the element" ϕ is NOT in $\{x\}$.

21. What is the cardinality of each of these sets?

a) $\{a\}$

b) $\{\{a\}\}$

Sol: a) $|\{a\}| = 1$, b) $|\{\{a\}\}| = 1$

29. a) $\{(a, y), (b, y), (c, y), (d, y), (a, z), (b, z), (c, z), (d, z)\}$ **b)** $\{(y, a), (y, b), (y, c), (y, d), (z, a), (z, b), (z, c), (z, d)\}$

30. What is the Cartesian product $A \times B$, where A is the set of courses offered by the mathematics department at a university and B is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.

Sol:

$A \times B = \{(c, p) \mid c$: course in Math department, p: professor in Math department$\}$
$c \in A$ and $p \in B$.

33. Let A be a set. Show that $\emptyset \times A = A \times \emptyset = \emptyset$.

$\emptyset \times A = \{(b, a) \mid b \in \emptyset$ and $a \in A\}$ and $\emptyset = \{\ \}$.

To show $\emptyset \times A = \emptyset$, we have to show

① $\emptyset \times A \supseteq \emptyset$: this is true because $\emptyset$ is a subset of any set.

② $\emptyset \times A \subseteq \emptyset$: given $(b, a) \in \emptyset \times A$ and assume $(b, a) \notin \emptyset$.
(contradiction)

since $(b, a) \notin \emptyset$, then it means $b \notin \emptyset$ and $a \notin \emptyset$

However, $b \notin \emptyset$ is a contradiction of $(b, a) \in \emptyset \times A$ (it implies $b \in \emptyset$)

Therefore, any given $(b, a) \in \emptyset \times A$ is also an element of $\emptyset$

and this proves $\emptyset \times A \subseteq \emptyset$.

Similarly, $A \times \emptyset = \emptyset$ by defining $A \times \emptyset = \{(a, b) \mid a \in A$ and $b \in \emptyset\}$

Thus, we have $\phi \times A = \phi = A \times \phi$.

37. How many different elements does $A \times B$ have if A has m elements and B has n elements?

Sol. Since $A \times B = \{(a,b) \mid a \in A \text{ and } b \in B\}$, then it totally has $n \cdot m$ combination which means $A \times B$ has nm elements