Section 1.6

5. Let w be "Randy works hard," let d be "Randy is a dull boy," and let j be "Randy will get the job." The hypotheses are w, $w \rightarrow d$, and $d \rightarrow \neg j$. Using modus ponens and the first two hypotheses, d follows. Using modus ponens and the last hypothesis, $\neg j$, which is the desired conclusion, "Randy will not get the job," follows.

6. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

Sol: Assume r: "It rains"
f: "It is foggy"
s: "sailing race will be held"
l: "lifesaving demonstration will go on"
t: "trophy will be awarded"

given premise #1 $\quad s \rightarrow t \quad \Leftarrow$ step 1 use the contraposition of premise #1

$\neg t \rightarrow \neg s$ premise #4

given premise #2 $\quad \neg t \quad \Leftarrow$ step 2 $\boxed{\text{premise \#4 \& given premise \#2 concludes } \neg s \text{ (premise \#5)}}$

premise #5 $\quad \neg s$

given premise #3 $\quad \neg r \vee \neg f \rightarrow s \wedge l \quad \Leftarrow$ step 3. use the contraposition of premise #3 $\quad \neg(s \wedge l) \rightarrow \neg(\neg r \vee \neg f)$

premise #6 $\quad \neg s \vee \neg l \rightarrow r \wedge f$

premise #7 $\quad r \wedge f \quad \Leftarrow$ step 4 premise #5 & premise #6 concludes $r \wedge f$

conclude $\therefore r \quad \Leftarrow$ step 5 premise #7 $\xrightarrow{\text{simplification}} r$.

19. **a)** Fallacy of affirming the conclusion **b)** Valid argument using modus tollens **c)** Fallacy of denying the hypothesis

20. Determine whether these are valid arguments.

a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

b) If $x^2 \neq 0$, where x is a real number, then $x \neq 0$. Let a be a real number with $a^2 \neq 0$; then $a \neq 0$.

Sol: a) Let $P(x)$: "x is a positive real number", $Q(x)$: "x^2 is a positive real number"

$$\cancel{\begin{array}{l} \forall x(P(x) \to Q(x)) \\ P(a) \to Q(a) \\ Q(a) \\ \hline \therefore P(a) \end{array}}$$

fallacy of affirming the conclusion

b) Let $P(x)$: "$x^2 \neq 0$, x is real", $Q(x)$: "$x \neq 0$"

$$\begin{array}{l} \forall x(P(x) \to Q(x)) \\ P(a) \to Q(a) \\ P(a) \\ \hline \therefore Q(a) \end{array}$$

This is valid which is modus ponens.

***35.** Determine whether this argument, taken from Kalish and Montague [KaMo64], is valid.

> If Superman were able and willing to prevent evil, he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

Sol: Let: a: " superman were able to prevent evil"

w: " superman were willing to prevent evil"

d: " superman would do so"

I: " superman would be impotent"

m: " superman would be malevolent"

e: " superman exists"

given premise #1 $a \wedge w \rightarrow d$

given premise #2 $\neg d$ (he doesn't prevent evil)

premise # 6 $\neg a \vee \neg w$ (step1 : premise#1 & premise #2 contraposition)

given premise #5 $e \rightarrow \neg I \wedge \neg m$

premise #8 $I \vee m \rightarrow \neg e$

conclusion $\therefore \neg e$

given premise# 3 $\neg a \rightarrow I$

given premise #4 $\neg w \rightarrow m$

premise #7 $I \vee m$ (step2: premise#3 & #6 $\rightarrow I$ premise#4 & #6 $\rightarrow m$)

⇐ step3 : contraposition of premise #5 $\neg(\neg I \wedge \neg m) \rightarrow \neg e$

⇐ step4: premise #7 & #8 conclude $\neg e$

Thus, this statement is valid.