Section 1.5

1. a) For every real number x there exists a real number y such that x is less than y. **b)** For every real number x and real number y, if x and y are both nonnegative, then their product is nonnegative. **c)** For every real number x and real number y, there exists a real number z such that $xy = z$. **3. a)** There is some student in your class who has sent a message to some student in your class. **b)** There is some student in your class who has sent a message to every student in your class. **c)** Every student in your class has sent a message to at least one student in your class. **d)** There is a student in your class who has been sent a message by every student in your class. **e)** Every student in your class has been sent a message from at least one student in your class. **f)** Every student in the class has sent a message to every student in the class.

10. Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody can fool Fred. a) $\forall x\, F(X, \text{Fred})$

b) Evelyn can fool everybody. b) $\forall x\, F(\text{Evelyn}, x)$

c) Everybody can fool somebody. c) $\forall x\, \exists y\, F(x, y)$

d) There is no one who can fool everybody. d) $\neg \exists x\, \forall y\, F(x, y)$

e) Everyone can be fooled by somebody. e) $\forall y\, \exists x\, F(X, y)$

f) No one can fool both Fred and Jerry.

f) $\forall x\, \neg(F(X, \text{Fred}) \wedge F(X, \text{Jerry}))$

or $\neg \exists x\, (F(X, \text{Fred}) \wedge F(X, \text{Jerry}))$

g) Nancy can fool exactly two people.

h) There is exactly one person whom everybody can fool.

i) No one can fool himself or herself.

j) There is someone who can fool exactly one person besides himself or herself.

10.

g) $\exists y_1 \exists y_2 \left(F(\text{Nancy}, y_1) \wedge F(\text{Nancy}, y_2) \wedge y_1 \neq y_2 \wedge \forall y \left(\underline{F(\text{Nancy}, y) \rightarrow (y = y_1 \vee y = y_2)} \right) \right)$

person 1 is not person 2

person 1

person 2

only person 1 & person 2.

h) $\exists y \forall x F(x, y) \wedge \forall z (\forall x F(x, y) \rightarrow z = y))$

i) $\neg \exists x F(x, x)$ or $\forall x \neg F(x, x)$

j) $\exists x \exists y \left(\underline{x \neq y} \wedge F(x, y) \wedge \forall z \left((F(x, z) \wedge \underline{z \neq x}) \rightarrow \underline{z = y} \right) \right)$

Not himself/herself

exactly one person.

27. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n \exists m (n^2 < m)$

b) $\exists n \forall m (n < m^2)$

c) $\forall n \exists m (n + m = 0)$

d) $\exists n \forall m (nm = m)$

e) $\exists n \exists m (n^2 + m^2 = 5)$

f) $\exists n \exists m (n^2 + m^2 = 6)$

a) True. Since "for each n there exists a m such that $n^2 < m$"

b) True. Since "There exists an n for all m such that $n < m^2$" (where $n = 0$)

c) True. Since "for each n there exists a m such that $n + m = 0$ (which is "$-n$")

d) True. Since "There exists an n for all m such that $nm = m$ (where $n = 1$)

e) True. Since "There exists an n and a m such that $n^2 + m^2 = 5$ (where $n = 1$, $m = 2$)

f) False. We can't find integers n and m such that $n^2+m^2=6$.

g) $\exists n \exists m(n+m=4 \wedge n-m=1)$

h) $\exists n \exists m(n+m=4 \wedge n-m=2)$

i) $\forall n \forall m \exists p(p=(m+n)/2)$

g) False. Since $\begin{cases} n+m=4 \\ n-m=1 \end{cases} \Rightarrow n=\frac{5}{2}, m=\frac{3}{2}$ which are NOT integers

h) True. since $\begin{cases} n+m=4 \\ n-m=2 \end{cases} \Rightarrow n=3, m=1$ ✓

i) False. Here is a counterexample: $n=2, m=3, p=\frac{n+m}{2}=\frac{5}{2}$ not an integer.

31. a) $\exists x \forall y \exists z \neg T(x, y, z)$ **b)** $\exists x \forall y \neg P(x, y) \wedge \exists x \forall y \neg Q(x, y)$ **c)** $\exists x \forall y (\neg P(x, y) \vee \forall z \neg R(x, y, z))$ **d)** $\exists x \forall y (P(x, y) \wedge \neg Q(x, y))$

33. a) $\exists x \exists y \neg P(x, y)$ **b)** $\exists y \forall x \neg P(x, y)$ **c)** $\exists y \exists x (\neg P(x, y) \wedge \neg Q(x, y))$ **d)** $(\forall x \forall y P(x, y)) \vee (\exists x \exists y \neg Q(x, y))$ **e)** $\exists x (\forall y \exists z \neg P(x, y, z) \vee \forall z \exists y \neg P(x, y, z))$