Section 1.4

1. **a)** T **b)** T **c)** F

2. Let $P(x)$ be the statement "The word x contains the letter a." What are these truth values?

a) P(orange) **b)** P(lemon)
c) P(true) **d)** P(false)

Sol a) P(orange) means : The word "orange" contains the letter "a" ⇒ T
b) P(lemon) ⇒ F
c) P(true) ⇒ F
d) P(false) ⇒ T.

4. State the value of x after the statement **if** $P(x)$ **then** $x := 1$ is executed, where $P(x)$ is the statement "$x > 1$," if the value of x when this statement is reached is

a) $x = 0$. **b)** $x = 1$.
c) $x = 2$.

Sol: a) "if $\underbrace{0>1}_{\text{false}}$, then $x:=1$" ⇒ X is still 0 after executed.
b) "if $\underbrace{1>1}_{\text{false}}$, then $x:=1$" ⇒ X is still 1 after executed.
c) "if $\underbrace{2>1}_{\text{true}}$, then $x:=1$" ⇒ X becomes 1 after executed

7. **a)** Every comedian is funny. **b)** Every person is a funny comedian. **c)** There exists a person such that if she or he is a comedian, then she or he is funny. **d)** Some comedians are funny. **9.** **a)** $\exists x(P(x) \wedge Q(x))$ **b)** $\exists x(P(x) \wedge \neg Q(x))$ **c)** $\forall x(P(x) \vee Q(x))$ **d)** $\forall x \neg(P(x) \vee Q(x))$ **11.** **a)** T **b)** T **c)** F **d)** F **e)** T **f)** F **13.** **a)** T **b)** T **c)** T **d)** T

19. **a)** $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$ **b)** $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$ **c)** $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$ **d)** $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$ **e)** $(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$

30. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

a) $\exists x\, P(x, 3)$ **b)** $\forall y\, P(1, y)$

c) $\exists y \neg P(2, y)$ **d)** $\forall x \neg P(x, 2)$

a) $\exists x\, P(x,3) \equiv P(1,3) \vee P(2,3) \vee P(3,3)$

b) $\forall y\, P(1,y) \equiv P(1,1) \wedge P(1,2) \wedge P(1,3)$

c) $\exists y \neg P(2,y) \equiv \neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)$

d) $\forall x \neg P(x,2) \equiv \neg P(1,2) \wedge \neg P(2,2) \wedge \neg P(3,2)$

38. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers. $x \in \mathbb{R}$

a) $\forall x(x^2 \neq x)$ **b)** $\forall x(x^2 \neq 2)$

c) $\forall x(|x| > 0)$

Sol: a) If $\boxed{x = 1}$, then $x^2 = x$ (which means there exists one real number "$x=1$" such that $x^2 = x$ and $\forall x(x^2 \neq x)$ is wrong)

b) If $\boxed{x = \sqrt{2} \text{ (or } x = -\sqrt{2})}$, then $x^2 = 2$ which makes $\forall x\,(x^2 \neq 2)$ a false statement.

c) If $\boxed{x = 0}$, then $|x| = 0 \not> 0$ which makes $\forall x\,(|x| > 0)$ a false statement.