Section 1.3

3. a)

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

b)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

4. Use truth tables to verify the associative laws

a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$.

Sol (a)

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Since $(p \vee q) \vee r$ and $p \vee (q \vee r)$ have the truth value, then

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

(b)

p	q	r	$p\wedge q$	$q\wedge r$	$(p\wedge q)\wedge r$	$p\wedge(q\wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Since $(p\wedge q)\wedge r$ and $p\wedge(q\wedge r)$ have the same truth value,
then $(p\wedge q)\wedge r \equiv p\wedge(q\wedge r)$.

6. Use a truth table to verify the first De Morgan law $\neg(p \wedge q) \equiv \neg p \vee \neg q$.

Sol

p	q	$p\wedge q$	$\neg(p\wedge q)$	$\neg p$	$\neg q$	$\neg p\vee\neg q$
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Since $\neg(p\wedge q)$ and $\neg p\vee\neg q$ have the same truth value,
then $\neg(p\wedge q)\equiv\neg p\vee\neg q$.

7. a) Jan is not rich, or Jan is not happy. **b)** Carlos will not bicycle tomorrow, and Carlos will not run tomorrow. **c)** Mei does not walk to class, and Mei does not take the bus to class. **d)** Ibrahim is not smart, or Ibrahim is not hard working.

8. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

b) Yoshiko knows Java and calculus.

c) James is young and strong.

d) Rita will move to Oregon or Washington.

Sol: a) It is $p \vee q$ where p is " Kwame will take a job in industry and q is "Kwame will go to graduate school.

Its negation is $\neg(p \vee q) \equiv \neg p \wedge \neg q$ which is

" Kwame will Not take a job in industy and will Not go to graduate school."

b) It is $p \wedge q$ where
p: Yoshiko Knows Java.
q Yoshiko Knows calculus.

Its negation is $\neg(p \wedge q) \equiv \neg p \vee \neg q$ which is

" Yoshiko does not know Java or calculus"

c) It is $p \wedge q$ where
p: James is young.
q: James is strong.

Its negation is $\neg(p \wedge q) \equiv \neg p \vee \neg q$ which is

" James is NOT young or strong."

d) It is $p \vee q$ where
p: Rita will move to Oregon.
q: Rita will move to Washington.

Its negation is $\neg(p \vee q) \equiv \neg p \wedge \neg q$ which is

"Rita will NOT move to Oregon and Washington."

11. a)

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

c)

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

b)

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

d)

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e)

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

f)

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

27. For $(p \to r) \wedge (q \to r)$ to be false, one of the two conditional statements must be false, which happens exactly when r is false and at least one of p and q is true. But these are precisely the cases in which $p \vee q$ is true and r is false, which is precisely when $(p \vee q) \to r$ is false. Because the two propositions are false in exactly the same situations, they are logically equivalent.

46. Suppose that a truth table in n propositional variables is specified. Show that a compound proposition with this truth table can be formed by taking the disjunction of conjunctions of the variables or their negations, with one conjunction included for each combination of values for which the compound proposition is true. The resulting compound proposition is said to be in **disjunctive normal form**.

Sol: If $n=2$, we have two propositions p and q and the truth table with 4 situations

	p	q		
①	T	T	$\to p \wedge q \equiv T$	— ①
②	T	F	$\to p \wedge \neg q \equiv T$	— ②
③	F	T	$\to \neg p \wedge q \equiv T$	— ③
④	F	F	$\to \neg p \wedge \neg q \equiv T$	— ④

The the compound proposition C of p and q can be found by the disjunction of true cases from the 4 situations

For example, if C is true at ①, ③, and ④, then

C is $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$

which is called disjunction normal form. (DNF)