MAT2440, Classwork9, Spring2025

ID:______________________________ Name:______________________________

1. Quantifier with **Restricted Domain**:

Given the following statements of quantifier with restricted domain:

(a) $\forall x < 0(x^2 > 0)$ (b) $\forall y \neq 0(y^3 \neq 0)$ (c) $\exists z > 0(z^2 = 2)$

What do the statements mean where the domain in each case consists of the real numbers?

(a) $\forall x<0(x^2>0)$ means "for every real number x with $x<0$, $x^2>0$"
or "for every negative real number x, $x^2>0$"
and it is true. $\boxed{\forall x(x<0 \to x^2>0)}$ conditional proposition

(b) $\forall y \neq 0 (y^3 \neq 0)$ means "for every nonzero y, $y^3 \neq 0$"
and it is true, $\boxed{\forall y(y \neq 0 \to y^3 \neq 0)}$

(c) $\exists z>0(z^2=2)$ means "there exists a positive real number z such that $z^2=2$"
and it is true, (which is $z=\sqrt{2}$) $\boxed{\exists z(z>0 \wedge z^2=2)}$

2. Observation:

Restriction of a universal quantification is the same as universal quantification of a conditional statement   example 1 (a) & (b)

Restriction of an existential quantification is the same as existential quantification of a conjunction "$\wedge$"   example 1 (c)

3. Quantifier over **Finite Domain**:

When the domain of a quantifier is finite (that is, when all its elements can be listed), quantifier statement can be expressed using propositional logic:

Let the elements of the domain be $x_1, x_2, x_3, \cdots, x_n$, then

$\forall x P(x)$ is the same as $P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots \wedge P(x_n)$

$\exists x P(x)$ is the same as $P(x_1) \vee P(x_2) \vee P(x_3) \vee \cdots \vee P(x_n)$

4. Given $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4. What is the truth value of $\forall x P(x)$?

Finite domain = 1, 2, 3, 4

$\forall x P(x)$ means "$P(1) \wedge P(2) \wedge P(3) \wedge P(4)$" and it is false.

$1^2<10 \quad 2^2<10 \quad 3^2<10 \quad 4^2<10$

$T \wedge T \wedge T \wedge F$

5. Given $P(x)$ be the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4. What is the truth value of $\exists x P(x)$?

Finite domain = 1, 2, 3, 4

$\exists x P(x)$ means "$P(1) \vee P(2) \vee P(3) \vee P(4)$" and it is true.

$1^2<10 \quad 2^2<10 \quad 3^2<10 \quad 4^2<10$

$T \vee T \vee T \vee F$

6. Given a proposition: "Every student in your class has taken a course in calculus."

x

(a) Using the **universal quantification** to express this proposition. $\forall x P(x)$

(b) Write down the negation of this proposition.

(c) Using the quantification to express the negation of this proposition.

(a) It's $\forall x P(x)$ where x: 'student in your class' (domain for x) $P(x)$: x has taken a course in calculus.

(b) $\neg \forall x P(x)$ means "It is not the case that every student in your class has taken calculus."

or "There is at least one student in your class who does not take calculus" $\Rightarrow \exists x (\neg P(x))$

(c) $\neg \forall x P(x) \equiv \exists x (\neg P(x))$

7. Given a proposition: "There is at least a student in your class who takes calculus."

$P(x)$

(a) Using the **existential quantification** to express this proposition.

(b) Write down the negation of this proposition.

(c) Using the quantification to express the negation of this proposition.

(a) $P(x)$: x takes calculus, x: student in your class (domain) $\Rightarrow \exists x P(x)$

(b) $\neg \exists x P(x)$ means "It is not the case that at least one student in your class takes calculus"

or "No one in your class takes calculus"

or "Every student in your class does not take calculus"

$\Rightarrow \forall x \neg P(x)$

(c) $\neg \exists x P(x) \equiv \forall x (\neg P(x))$