

MAT2440, Classwork8, Spring2025

ID: _____ Name: _____

1. Quantifications:

To create a proposition from a propositional function, we can use quantification, which express that a predicate is true over a range of elements. In English, all, some, many, none, and few are used for quantifications.

All students are here.

Some people wear blue today.

2. The Universal Quantifier '∀':

' $\forall x$ ' means for all x / for each x (in the domain).

The **universal quantifications** of $P(x)$, denoted by the notation $\forall x P(x)$, is the statement $P(x)$ for all value of x in the domain

and ' $\forall x P(x)$ ' is read as for all x , $P(x)$.

An element for which $P(x)$ is **false** is called a Counterexample to $\forall x P(x)$.

3. If $N(x)$ is "Computer x is connected to the network" and the domain consists of all computers on campus. What does the statement ' $\forall x N(x)$ ' mean?

$\forall x N(x)$ means "for every computer x on campus, the computer x is connected to the network"

or "Every computer on campus is connected to the network"

4. Let $P(x)$ be " $x + 1 > x$ ". What is the truth value of ' $\forall x P(x)$ ', where the domain consists of all real numbers?

$\forall x P(x)$ means "for all real number x , $x+1 > x$ "

and it is true. ($\forall x P(x)$ is true)

5. Determine the truth value of ' $\forall x(x^2 > 0)$ '.

This means "for all x , x^2 is greater than 0".

This is not true, because when $x=0 \Rightarrow x^2=0$
 \uparrow
 counterexample
 and this statement is false

6. The Existential Quantifier ' \exists ':

' $\exists x$ ' means "There exists an x (in the domain)"

The **existential quantifications** of $P(x)$, denoted by the notation $\exists x P(x)$, is the statement there exists an element x in the domain such that $P(x)$ and

' $\exists x P(x)$ ' is read as there exists an x such that $P(x)$.

↙ There is at least one x

7. ◦ For some x ,

Statement	When True?	When False?
$\forall x P(x)$	$P(x)$ is true for all x	There is an x for which $P(x)$ is false
$\exists x P(x)$	There is an x for which $P(x)$ is true.	$P(x)$ is false for all x

8. What is the meaning of ' $\exists x(x > 3)$ ' when the domain consists of all real number?

$\exists x (x > 3)$ means "there exists a real number x such that $x > 3$ "
 $\underbrace{x > 3}$
 \downarrow propositional function

9. What is the truth value of ' $\exists x(x > 3)$ '?

From \mathcal{R} , we have the proposition $\exists x (x > 3)$ and it is true.