MAT2440, Classwork7, Spring2025

ID:______________________________ Name:______________________________

1. Use identities to prove "$\neg(p \longrightarrow q) \equiv p \wedge \neg q$"

~~$\neg(p \to q) \equiv p \wedge (\neg q)$~~

$$\neg(p \to q) \equiv \neg(\neg p \vee q)$$ first one in Group II

$$\equiv \neg(\neg p) \wedge \neg q$$ De Morgan's law

$$\equiv p \wedge (\neg q)$$ Double Negation

2. Use identities to prove "$(p \wedge q) \longrightarrow (p \vee q)$" is a tautology.

$$(p \wedge q) \to (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$$

$$\equiv (\neg p \vee \neg q) \vee (p \vee q)$$ De Morgan's

$$\equiv (\neg p \vee p) \vee (\neg q \vee q)$$ associative and commutive

$$\equiv T \vee T$$ negation

$$\equiv T$$ domination

3. Group III of the logically equivalences: Identities related to biconditional statements.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ ← "definition" of "$p \leftrightarrow q$"

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

(3) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Prove "$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$"

Proof: $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

Group II, the 1st one

$$\equiv ((\neg p \wedge \neg q) \vee (q \wedge \neg q)) \vee ((\neg p \wedge p) \vee (q \wedge p))$$

negation law

$$\equiv ((\neg p \wedge \neg q) \vee F) \vee (F \vee (q \wedge p))$$

Identity law

$$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$$

$$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

prove $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

proof: Group III (3)

$$\neg(p \leftrightarrow q) \equiv \neg((p \wedge q) \vee (\neg p \wedge \neg q))$$

De Morgan's

$$\equiv \neg(p \wedge q) \wedge \neg(\neg p \wedge \neg q)$$

De Morgan's

$$\equiv (p \to \neg q) \wedge (p \vee q)$$

$\neg(p \to \neg q) \equiv p \wedge q$ and negation in Group II

$$\equiv (p \to \neg q) \wedge (p \vee \neg(\neg q))$$

$$\equiv (p \to \neg q) \wedge (\neg q \to p)$$

(Group II: $p \to q \equiv (\neg p) \vee q$)

$$\equiv p \leftrightarrow \neg q$$ (Group III first one)

4. Predicate logic and Propositional function:

The predicate logic allows variables in propositions and enables us to reason and explore relationships between objects. A propositional function is a statement with variables and has been used on predicate logic. Once the values have been assigned to the variables, the propositional function becomes a proposition, and has truth value

5. Let $P(x)$ denote the statement ''$x > 3$''. What are the truth values of $P(2)$ and $P(4)$.

P(x) denotes " x is greater than 3"

x: variable (subject)

is greater than 3: predicate (property)

P(2) is "2 > 3" false

P(4) is "4 > 3" True

6. Let $Q(x, y)$ denote the statement ''$x = y + 3$''. What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$.

Q(1,2) means "1 = 2+3" which is false. ($y=2$, $x=1$)

Q(3,0) means "3 = 0+3" which is true. ($y=0$, $x=3$)

7. Given a computer programing ''If $x > 0$, then $x := x + 1$''. Using the terminology of propositional function to explain it.

this is P(x)

If P(x) is true, then x is increased by 1.

If P(x) is false, then x is not changed.