MAT2440, Classwork6, Spring2025

ID:________________________________ Name:__________________________________

1. Group I of the logically equivalences: Identities related to '¬', '∨', '∧'.

Identity laws	$p \wedge \mathbf{T} \equiv$ p $p \vee \mathbf{F} \equiv$ p	Domination laws	$p \vee \mathbf{T} \equiv$ T $p \wedge \mathbf{F} \equiv$ F
Idempotent laws	$p \vee p \equiv$ p $p \wedge p \equiv$ p	Negation laws	$p \vee \neg p \equiv$ T $p \wedge \neg p \equiv$ F
Double Negation laws	$\neg(\neg p) \equiv$ p	Commutative laws	$p \vee q \equiv$ $q \vee p$ $p \wedge q \equiv$ $q \wedge p$
Associate laws	$(p \vee q) \vee r \equiv$ $p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv$ $p \wedge (q \wedge r)$	Distributive laws	$p \vee (q \wedge r) \equiv$ $(p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv$ $(p \wedge q) \vee (p \wedge r)$
Absorption laws	$p \vee (p \wedge q) \equiv$ p $p \wedge (p \vee q) \equiv$ p	De Morgan's laws	$\neg(p \wedge q) \equiv$ $\neg p \vee \neg q$ $\neg(p \vee q) \equiv$ $\neg p \wedge \neg q$

2. Prove one of the De Morgan's laws: "$\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$" by using truth table.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$(\neg p) \wedge (\neg q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

3. Show that "$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$"

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$ De Morgan's law

$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$ De Morgan's law

$\equiv \neg p \wedge (p \vee \neg q)$ Double Negation

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ Distributive law

$\equiv F \vee (\neg p \wedge \neg q)$ Negation law

$\equiv (\neg p \wedge \neg q) \vee F$ commutative $\equiv \neg p \wedge \neg q$ identity law

4. Using De Morgan's laws to express the negation of the given proposition:

''Miguel has a cellphone, and he has a laptop''

$p \wedge q$

negation: $\neg(p \wedge q) = \neg p \vee \neg q$

Miguel doesn't have a cell phone or he doesn't have a laptop

5. Group II of the logically equivalences: Identities related to conditional statements.

$p \to q \equiv (\neg p) \vee q$ $\neg p \to q \equiv p \vee q$ $\neg(p \to q) \equiv p \wedge (\neg q)$ $\neg(p \to \neg q) \equiv p \wedge q$	$p \to q \equiv \neg q \to \neg p$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$ $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$	$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$ $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

6. Use the truth table to prove ''$p \to q \equiv (\neg p) \vee q$''

p	q	$p \to q$	$\neg p$	$(\neg p) \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

since they have the same true value

$p \to q \equiv (\neg p) \vee q$