MAT2440, Classwork5, Spring2025

ID:_____ Name:_____

1. Translating Sentence into expression with propositional variables and logical connectives:

"The automated reply cannot be sent when the file system is full."

f: the file system is full.

A: The automated reply can be sent $\Rightarrow \neg A$

$\Rightarrow f \rightarrow \neg A$

2. Translating Sentence into expression with propositional variables and logical connectives:

"You can access the Internet from campus only ~~of~~ if you are a computer science major or you are not a freshman."

p: You can access the internet

q: You're a computer science major

r: you are not a freshman

$p \rightarrow (q \vee r)$

3. Translating Sentence into expression with propositional variables and logical connectives:

"You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old."

p: You cannot ride the roller coaster

q: You are under 4 feet tall

r: You are older than 16.

$(q \wedge \neg r) \rightarrow p$

p if q or p unless r

$(q \rightarrow p) \vee (\neg r \rightarrow p)$

$(q \wedge \neg r) \rightarrow p$

4. Definition of **Tautology**, **Contradiction**, and **Contingency**:

A compound proposition that is always **true** is called a tautology .

A compound proposition that is always **false** is called a contradiction .

A compound proposition is neither **a tautology** nor **a contradiction** is called a contingency

5. Show that ‘‘$p \vee \neg p$’’ is a tautology and ‘‘$p \wedge \neg p$’’ is a contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

$p \wedge \neg p$ is a contradiction.

$p \vee \neg p$ is a tautology

6. Definition of **Logical Equivalences**:

The compound propositions p and q called logical equivalent if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes p and q are logically equivalent.

7. Using the logically equivalent symbol ‘≡’ to rewire the following expressions:

(a) ‘‘$p \vee \neg p$’’ is a tautology. $p \vee \neg p \equiv T$

(b) ‘‘$p \wedge \neg p$’’ is a contradiction. $p \wedge \neg p \equiv F$

(c) ‘‘$p \rightarrow q$’’ and ‘‘$\neg q \rightarrow \neg p$’’ have the same truth value.

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

(d) ‘‘$q \rightarrow p$’’ and ‘‘$\neg p \rightarrow \neg q$’’ have the same truth value.

$q \rightarrow p \equiv \neg p \rightarrow \neg q$