

MAT2440, Classwork3, Spring2025

ID: _____ Name: _____

1. Find the **Converse**, the **Contrapositive**, and the **Inverse** of the given conditional statement.

“The New York Yankees wins whenever it is raining.”

$P \rightarrow Q$; If it is raining, then the Yankees wins.

Converse $Q \rightarrow P$; If the Yankees wins, then it is raining

Contrapositive: If the Yankees doesn't win, then it is NOT raining

Inverse: If it is not raining, then the Yankees doesn't win.

2. The truth table of the **Converse**, **Inverse**, and **Contrapositive** of a conditional statement:

p	q	$p \rightarrow q$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T	T	T	F	F	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

3. Using the truth table from 2. to answer the following questions.

(a) What do you observe from the results between “ $p \rightarrow q$ ” and “ $\neg q \rightarrow \neg p$ ”?

(b) What do you observe from the results between “ $q \rightarrow p$ ” and “ $\neg p \rightarrow \neg q$ ”?

(a) “ $p \rightarrow q$ ” is equivalent to “ $\neg q \rightarrow \neg p$ ”.
 (Original) means “same truth value” (contrapositive)

(b) “ $q \rightarrow p$ ” is equivalent to “ $\neg p \rightarrow \neg q$ ”
 (converse) (Inverse)

(a) “If you go, I will go” \Leftrightarrow “If I don't go, you won't go”
 equivalent to

4. Definition of the **Biconditional statement** for two propositions:

Let p and q be two propositions. The biconditional statement " $p \leftrightarrow q$ " is the proposition " p if and only if q ."

Biconditional statements are also called bi-implication.

$$\left. \begin{array}{l} p \text{ if } q : q \rightarrow p \\ p \text{ only if } q : p \rightarrow q \end{array} \right\} p \text{ if and only if } q \text{ (iff)}$$

5. The truth table for the biconditional statement " $p \leftrightarrow q$ ":

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$	$p \leftrightarrow q$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

" $p \leftrightarrow q$ " is **true** when p and q have the Same truth values and is **false** otherwise.

6. Other ways to express the biconditional statements:

" p is sufficient and necessary for q ."

"if p then q , and conversely."

" p exactly when q ."

$$\left. \begin{array}{l} \text{"}p \text{ is sufficient and necessary for } q\text{"} \\ \text{"if } p \text{ then } q, \text{ and conversely.} \\ \text{"}p \text{ exactly when } q\text{"} \end{array} \right\} p \leftrightarrow q$$

7. Given two propositions p and q . Find " $p \leftrightarrow q$ ".

p : "You can take the flight."; q : "You buy a flight ticket."

$p \leftrightarrow q$: You can take the flight if and only if you buy a flight ticket