MAT2440, Classwork38, Spring2025

ID:________________________________ Name:________________________________

1. Theorem for Representation of Integers. $(281)_{10} = 2\cdot 10^2 + 8\cdot 10^1 + 1\cdot 10^0$

Let $b > 1$ be an integer. Then if n is a positive integer, it can be expressed uniquely in the form

$$n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b^1 + a_0 b^0,$$

where $k \geq 0$ is an integer, $a_0, a_1, \cdots, a_{k-1}, a_k$ are integers with $0 \leq a_0, a_1, \cdots, a_{k-1} \leq b$, and $0 < a_k \leq b$.

We called this representation of n **base** $\underline{b}$ **expansion of** $\underline{n}$, denoted by $\underline{(a_k a_{k-1} a_{k-2} \cdots a_0)_b}$.

2. Converting Decimals to Integers of Other Bases.

(a) Find the octal expansion of $(12345)_{10}$. $= (30071)_8$

$12345 \div 8 =$ quotient $\underline{1543}$ with remainder $\underline{1}$ ➡ $12345 = \underline{1543} \times 8 + \underline{1}$.

$\underline{1543} \div 8 =$ quotient $\underline{192}$ with remainder $\underline{7}$ ➡ $\underline{1543} = \underline{192} \times 8 + \underline{7}$.

$\underline{192} \div 8 =$ quotient $\underline{24}$ with remainder $\underline{0}$ ➡ $\underline{192} = \underline{24} \times 8 + \underline{0}$.

$\underline{24} \div 8 =$ quotient $\underline{3}$ with remainder $\underline{0}$ ➡ $\underline{24} = \underline{3} \times 8 + \underline{0}$.

$\underline{3} \div 8 =$ quotient $\underline{0}$ with remainder $\underline{3}$ ➡ $\underline{3} = \underline{0} \times 8 + \underline{3}$.

Therefore, we have

$12345 = 1534 \times 8 + 1$

$= (192 \times 8 + 7) \times 8 + 1 = 192 \times 8^2 + 7 \times 8^1 + 1$

$= (24 \times 8 + 0) \times 8^2 + 7 \times 8^1 + 1 = 24 \times 8^3 + 0 \times 8^2 + 7 \times 8^1 + 1$

$= (3 \times 8 + 0) \times 8^3 + 0 \times 8^2 + 7 \times 8^1 + 1$

$= 3 \times 8^4 + 0 \times 8^3 + 0 \times 8^2 + 7 \times 8^1 + 1 \times 8^0 = (30071)_8$

(b) Find the hexadecimal expansion of $(117730)_{10}$.

$117730 \div 16 \Rightarrow Q\ \underline{7358}\quad R\ \underline{2}$

$\underline{7358} \div 16 \Rightarrow Q\ \underline{459}\quad R\ \underline{14} \to E$

$459 \div 16 \Rightarrow Q\ \underline{28}\quad R\ \underline{11} \to B$

$28 \div 16 \Rightarrow Q\ \underline{1}\quad R\ \underline{12} \to C$

$1 \div 16 \Rightarrow Q\ \underline{0}\quad R\ \underline{1}$

Stop when quotient $= 0$

$(117730)_{10} = (1CBE2)_{16}$

2. (c) Find the binary expansion of $(241)_{10}$.

$241 \div 2 = Q\ \underline{120}\quad R\ 1$

$120 \div 2 = Q\ \underline{60}\quad R\ 0$

$60 \div 2 = Q\ \underline{30}\quad R\ 0$

$30 \div 2 = Q\ \underline{15}\quad R\ 0$

$15 \div 2 = Q\ \underline{7}\quad R\ 1$

$7 \div 2 = Q\ \underline{3}\quad R\ 1$

$3 \div 2 = Q\ \underline{1}\quad R\ 1$

$1 \div 2 = Q\ \underline{0}\quad R\ 1$

stop at quotient = 0

$\Rightarrow (241)_{10} = (1111\ 0001)_2$

3. Conversion between Binary, Octal, and Hexadecimal Expansions.

TABLE 1 **Hexadecimal, Octal, and Binary Representation of the Integers 0 through 15.**

Decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hexadecimal	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Octal	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

$1\cdot 2^1 + 0\cdot 2^0 = 2$

4. Convert $(11\ 1110\ 1011\ 1100)_2$ to octal and hexadecimal expansions.

Octal: $(011\ 111\ 010\ 111\ 100)_2 = (3\ 7\ 2\ 7\ 4)_8$

in base 10 $0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 = 3$

$1\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 = 7$

hexadecimal

$(0011\ 1110\ 1011\ 1100)_2 = (3\ E\ B\ C)_{16}$

$0\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 1\cdot 2^0 = 3$

$1\cdot 2^3 + 1\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 = 8+4+2+0 = 14$

5. Find the binary expansion of $(756)_8$ and $(A8D)_{16}$.

$(7\ 5\ 6)_8 \to (111\ 101\ 110)_2$

$(A\ 8\ D)_{16}$

Base 10: 10, 8, 13

$10 = 8+2 = 1\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0 \to 1010$

$8 = 1\cdot 2^3 + 0\cdot 2^2 + 0\cdot 2^1 + 0\cdot 2^0 \to 1000$

$13 = 1\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0 \to 1101$

$\Rightarrow (1010\ 1000\ 1101)_2$