MAT2440, Classwork37, Spring2025

ID:____________________ Name:____________________

1. Find all integers that are congruent to 3 modulo 5.

Sol: $a \equiv 3 \pmod 5 \Rightarrow a = 5\cdot n + 3$

$n=0,\ a = 5\cdot 0 + 3 = 3$

$n=1,\ a = 5\cdot 1 + 3 = 8$

$n=2,\ a = 5\cdot 2 + 3 = 13$

$\vdots$

$n=-1,\ a = 5\cdot(-1) + 3 = -2$

$n=-2,\ a = 5\cdot(-2) + 3 = -7$

$\vdots$

$a = \{ \ldots, -7, -2, 3, 8, 13, 18, \ldots \}$

+5 +5 +5 +5 +5 . . .

2. Find the integer a such that $a \equiv 3 \pmod{12}$ and $11 \le a \le 22$.

Sol $a \equiv 3 \pmod{12} \Rightarrow a = 12\cdot n + 3$

if $11 \le a \le 22$, then $11 \le 12\cdot n + 3 \le 22$
$\quad -3 \qquad -3 \qquad -3$

$\Rightarrow 8 \le 12n \le 19$

$\Rightarrow \frac{8}{12} \le n \le \frac{19}{12} \Rightarrow 0 < n \le 1$

$n=1,\ a = 12\cdot 1 + 3 = 15 \rightarrow a = 15.$

$(n=0,\ a = 12\cdot 0 + 3 = 3)$

3. Find all integers between -50 and 50 that are congruent to 6 modulo 11.

Sol: if $a \equiv 6 \pmod{11}$ and $-50 \le a \le 50$, we have

$a = 11\cdot n + 6$ and $-50 \le 11\cdot n + 6 \le 50$
$\quad -6 \qquad -6 \qquad -6$

$\Rightarrow -56 \le 11n \le 44 \Rightarrow \frac{-56}{11} \le n \le \frac{44}{11}$

$\Rightarrow -5 \le n \le 4.$

$n=-5,\ a = 11\cdot(-5) + 6 = -49$ +11

$n=-4,\ a = 11\cdot(-4) + 6 = -38$ +11

$n=-3,\ a = 11\cdot(-3) + 6 = -27$ +11

$n=-2,\ a = 11(-2) + 6 = -16$ +11

$n=-1,\ a = 11(-1) + 6 = -5$

$n=0,\ a = 11\cdot 0 + 6 = 6$ +11

$n=1,\ a = 11\cdot 1 + 6 = 17$ +11

$n=2,\ a = 11\cdot 2 + 6 = 28$ +11

$n=3,\ a = 11\cdot 3 + 6 = 39$ +11

$n=4,\ a = 11\cdot 4 + 6 = 50$

1. Representation of Integers.

base 10

In everyday life, we use decimal notation to represent numbers. However, computers usually use binary notation, octal notation, and hexadecimal notation when expressing characters.

base 2 base 8 base 16

2. Decimal notation (base 10):

Acceptable digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

For example, $2845 = (2845)_{10} = 2000 + 800 + 40 + 5$

$= 2 \times 10^3 + 8 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$.

3. Binary notation (base 2): Acceptable digits: 0, 1.

What is the decimal expansion of the number with binary expansion $(10101)_2$?

$(10101)_2 = 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 1 \times 16 + 0 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1$

$= 21$ $(21)_{10}$

4. Octal notation (base 8): Acceptable digits: 0, 1, 2, 3, 4, 5, 6, 7.

What is the decimal expansion of the number with octal expansion $(7016)_8$?

$(7016)_8 = 7 \times 8^3 + 0 \times 8^2 + 1 \times 8^1 + 6 \times 8^0$

$= 7 \times 512 + 0 \times 64 + 1 \times 8 + 6 \times 1$

$= 3598$

5. Hexadecimal notation (base 16):

Acceptable digits:

10 11 12 13 14 15

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

What is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$?

$(2AE0B)_{16} = 2 \times 16^4 + A \times 16^3 + E \times 16^2 + 0 \times 16^1 + B \times 16^0$

$= 2 \times 16^4 + 10 \times 16^3 + 14 \times 16^2 + 0 \times 16^1 + 11 \times 1$

$= 2 \times 65536 + 10 \times 4096 + 14 \times 256 + 0 + 11$

$= 175627$

The above examples show the meaning of integer representation of different bases as well as how to convert them to decimal.