MAT2440, Classwork36, Spring2025

ID:________________________ Name:________________________

1. Definition of Division.

If $a, b \in \mathbb{Z}$ with $a \neq 0$ and there is a $c \in \mathbb{Z}$ such that $\frac{b}{a} = c$, we say that a divides b.

When a divides b, denoted by $a \mid b$, we say that a is a factor or divisor of b, and that b is a multiple of a. Otherwise, $a \nmid b$ when a does not divide b.

2. Determine whether (a) 3 divides 12, (b) 3 divides 7.

(a) Yes, 3 divides 12 $\left(\frac{12}{3} = 4\right)$

$3 \mid 12$

$12 = 3 \cdot \underline{4}$ quotient

(b) NO, 3 does not divide 7

$3 \nmid 7$

$7 = 3 \cdot \underline{2} + \underline{1}$ remainder

quotient

3. The Division Algorithm.

Let a be an integer and d a positive integer. Then there are unique integers q and r, with $0 \leq r < d$, such that $a = dq + r$.

4. In 3., when $a = dq + r$, d is called the divisor, a is called the dividend, q is called the quotient, denoted by a div d, and r is called the remainder, denoted by a mod d.

5. In 4., if "a **mod** $d = 0$", what can we say about "a"? $(a = dq + r)$

$\Leftrightarrow r = 0 \Leftrightarrow a = dq \Leftrightarrow d \mid a$ (d divides a)

6. What are the quotient and remainder when 11 is divides by 3?

$11 = 3 \times \underline{3} + 2$ remainder

quotient

11 div 3 = 3

11 mod 3 = 2

7. What are the quotient and remainder when -11 is divides by 3?

$\cancel{-11 = 3 \times (-3) + (-2)}$

quotient remainder

$-11 = 3 \times \underline{(-4)} + 1$

$\cancel{-11 = 3 \times (-5) + 4}$

-11 div $3 = -4$

-11 mod $3 = 1$

8. The Quotient and Remainder with the Floor Function: In 3., when $a = dq + r$, we have

$$q = a \textbf{ div } d = \left\lfloor \frac{a}{d} \right\rfloor \text{ and } r = a \textbf{ mod } d = a - dq = a - d\left\lfloor \frac{a}{d} \right\rfloor$$

9. Find (1) -37 **div** 7, (2) -37 **mod** 7, (3) 51 **mod** 6, and (4) -51 **mod** 6.

(1) $-37 = 7 \times (-6) + 5$

$\cancel{7 \times (-5) + (-2)}$

$\cancel{7 \times (-7) + 12}$

$-37 \text{ div } 7 = -6$

(2)

$-37 \text{ mod } 7 = 5$

(3) $51 = 6 \times 8 + 3$

$51 \text{ mod } 6 = 3$

(4) $-51 = 6 \times (-9) + 3$

$-51 \text{ mod } 6 = 3$

10. Definition of Congruent and Modulo.

Let a, b be integers and m be a positive integer. Then a is congruent to b modulo m if m divides $a-b$, that is,

② $a \equiv b \pmod{m}$ if and only if $m \mid (a-b)$.

We say that "$a \equiv b \pmod{m}$" is a congruence and m is its modulus.

Otherwise, if a is *not congruent* to b *modulo* m, we write $a \not\equiv b \pmod{m}$.

$51 \text{ mod } 6 = 3$

$-51 \text{ mod } 6 = 3$ $\Rightarrow -51 \equiv 51 \pmod{6} \Rightarrow 6 \mid (51-(-51))$

11. Let a, b be integers and m be a positive integer. We have

① $a \equiv b \pmod{m}$ if and only if $a \text{ mod } m = r = b \text{ mod } m$.

relation

equation

12. Determine whether 17 is congruent to 5 modulo 6.

① (from 11)

$17 \text{ mod } 6 = 5$

$(17 = 6 \times \underset{\text{quotient}}{2} + \boxed{5})$ remainder

$5 \text{ mod } 6 = 5$

$(5 = 6 \times 0 + 5)$

$\Rightarrow 17 \equiv 5 \pmod{6}$

② (from 10.)

$17 - 5 = 12$

$6 \mid 12$