MAT2440, Classwork32, Spring2025

ID:_____ Name:_____

1. The order of growth for functions: **Exponential functions**.

$$b^x, b > 1$$

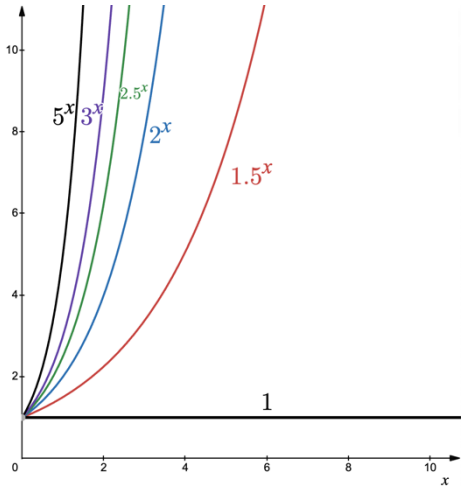
For each $x > 0$, we have

$$\cdots < 1.5^x < 2^x < 2.5^x < 3^x < 5^x < \cdots.$$

For each $x >$ certain positive number, we have

$$x^b < b^x$$

For example, $x^2 < 2^x$ for $x > 4$.



2. Find the big-O for $f(x) = x^2 + 2^x$.

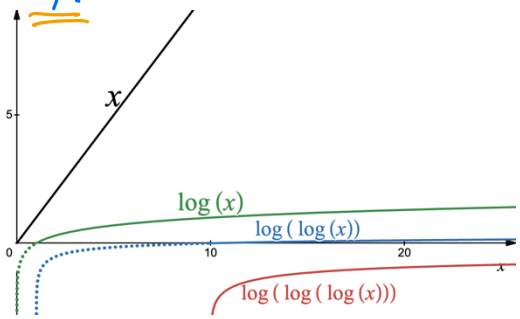
upper bound

$f(x) = x^2 + 2^x \le 2^x + 2^x = 2 \cdot (2^x)$ for $x > 4$

$\Rightarrow f(x)$ is $O(2^x)$ with $C = 2$, $k = 4$.

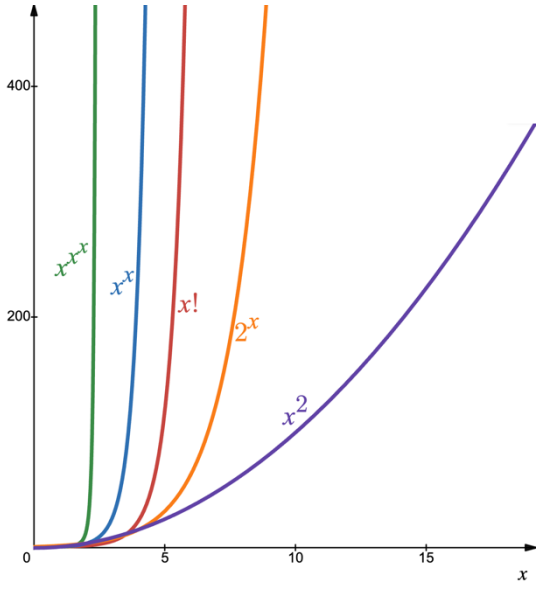
3. The order of growth for functions: **Logarithmic functions**.

$$\log(\log(\log(x))) < \log(\log(x)) < \log(x) \ll x^p$$



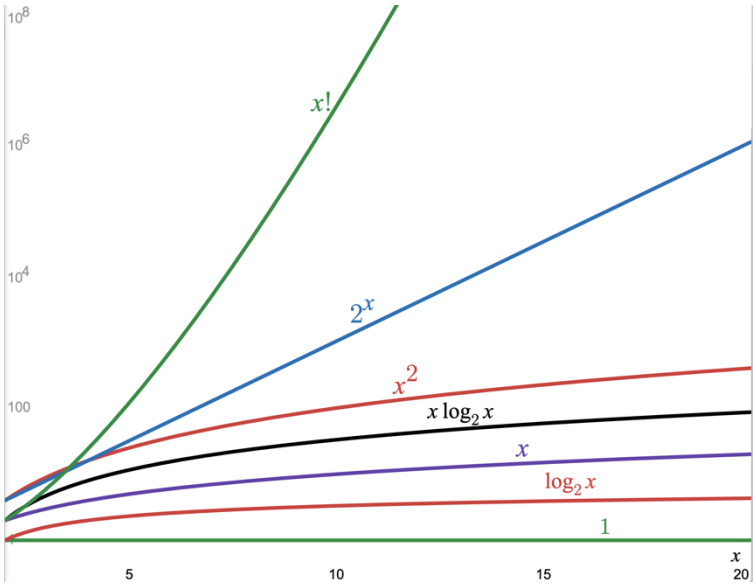
4. Other growth functions.

$$x^p < b^x < x! < x^x < x^{x^x}$$



5. The often-used list of the big-O relationship.

$$1 \underline{\prec} \log(x) \underline{\prec} x \underline{\prec} x\log(x) \underline{\prec} x^2 \underline{\prec} 2^x \underline{\prec} x!$$



6. Arrange the functions $8\sqrt{n}$, $(\log(n))^2$, $2n \cdot \log(n)$, $n!$, $(1.1)^n$, and n^2 in a list so that each function is the big-O of the next function.

$(\log(n))^2$, $8\sqrt{n}$, $2n\cdot\log(n)$, n^2, $(1.1)^n$, $n!$

7. Find the least integer n such that $f(x) = \frac{x^5+2x^3}{2x^2+1}$ is $O(x^n)$. → $n=3$

$f(x) = \frac{x^5+2x^3}{2x^2+1} \le \frac{x^5+2x^3}{2x^2} \le \frac{x^5+2\cdot x^5}{2x^2} \le \frac{3x^5}{2x^2} = \frac{3}{2}x^3$

$\Rightarrow f(x)$ is $O(x^3)$ with $C=\frac{3}{2}$ and $k=1$ $(x>1)$

8. Give a big-O estimate for $f(x) = (x+1)\cdot\log(x^2+1) + 3x^2$.

$\log(x^2+1) \le \log(x^2+x^2) = \log(2x^2)$

$= \log 2 + \log x^2 = \log 2 + 2\log x \le \log x + 2\log x = 3\log x$ if $x>2$.

Then $f(x) = (x+1)\cdot\log(x^2+1) + 3x^2$ ($1 \le x$)

$\le (x+1)\cdot 3\log x + 3x^2 \le (x+x)\cdot 3\log x + 3x^2$

$= 6x\log x + 3x^2 \le 6x\cdot x + 3x^2 = 9x^2$ if $x>2$ ($\log x \le x$)

$\Rightarrow f(x)$ is $O(x^2)$ with $C=9$ and $k=2$