MAT2440, Classwork31, Spring2025

ID:________________________________ Name:__________________________________

1. Use the cashier's algorithm to find the combination of 89 cents:

$c_1 = 25$ $c_2 = 10$ $c_3 = 5$ $c_4 = 1$

$$89\ cents = ?\ quarters + ?\ dimes + ?\ nickels + ?\ pennies.$$

Initialization: $r = 4$, $i = 1$ to 4, $n = 89$ (cents)

$n \geq c_i$

$i = 1$ $d_1 = 0$ $89 \geq 25$ Yes $d_1 = 1$ $n = 89 - 25 = 64$

$d_1 = 1$ $64 \geq 25$ Yes $d_1 = 2$ $n = 64 - 25 = 39$

$d_1 = 2$ $39 \geq 25$ Yes $d_1 = 3$ $n = 39 - 25 = 14$

$\boxed{d_1 = 3}$ $14 \geq 25$ NO STOP

$i = 2$ $d_2 = 0$ $14 \geq 10$ Yes $d_2 = 1$ $n = 14 - 10 = 4$

$\boxed{d_2 = 1}$ $4 \geq 10$ NO STOP

$i = 3$ $\boxed{d_3 = 0}$ $4 \geq 5$ NO STOP

$i = 4$ $d_4 = 0$ $4 \geq 1$, Yes, $d_4 = 1$ $n = 4 - 1 = 3$

$d_4 = 1$ $3 \geq 1$, Yes $d_4 = 2$ $n = 3 - 1 = 2$

$d_4 = 2$ $2 \geq 1$ Yes $d_4 = 3$ $n = 2 - 1 = 1$

$d_4 = 3$ $1 \geq 1$ Yes $d_4 = 4$ $n = 1 - 1 = 0$

$\boxed{d_4 = 4}$ $0 \geq 1$ NO STOP

Return $d_1 = 3$, $d_2 = 1$, $d_3 = 0$, $d_4 = 4$

$(89 = 3 \times 25 + 1 \times 10 + 0 \times 5 + 4 \times 1)$

2. The Growth of Function: An Introduction.

In 3.3, we will study the **number of operators** used by those algorithms in 3.1:

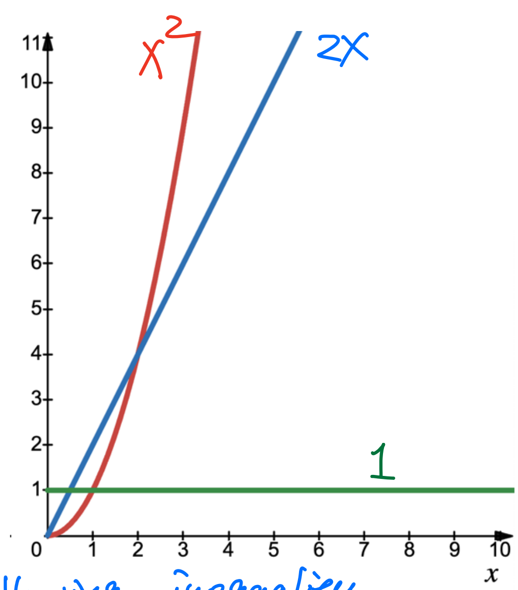
In linear search of a number in the list of $\{2, 3, 5, 7\}$, you need (at most) __4__ comparisons.

But if the length of the list becomes 10,000, then you need (at most) __10,000__ comparisons.

This implies that the number of operators __increase__ when the length of the list "n" increases and different algorithms have different rate of growth, which is measured by **the time complexity** of the algorithm.

3. Given $f(x) = x^2 + 2x + 1$. What happens when $x \to \infty$?

	x^2 +	$2x$ +	1
$x=1$	1	2	1
$x=10$	100	20	1
$x=100$	10000	200	1
	grows fast	grows slow	doesn't grow



$x \to \infty$ $x^2 >> 2x >> 1$

we can express the above observation as the following inequality

$$x^2 + 2x + 1 \leq x^2 + 2x^2 + x^2 = 4x^2 \quad \text{when } x > 1$$

4. Definition of **Big-*O*** Notation.

Let $f(x)$ and $g(x)$ be functions. We say that $f(x)$ is $O(g(x))$ ("$f(x)$ is big-oh of $g(x)$") if there are constants C and k such that

$$0 \le f(x) \le \underline{C \cdot g(x)} \text{ whenever } x > \underline{k}.$$

The constants C and k are called __witnesses__ to the relationship $f(x)$ is $O(g(x))$. To establish this relationship, we need only __one__ pair of C and k.

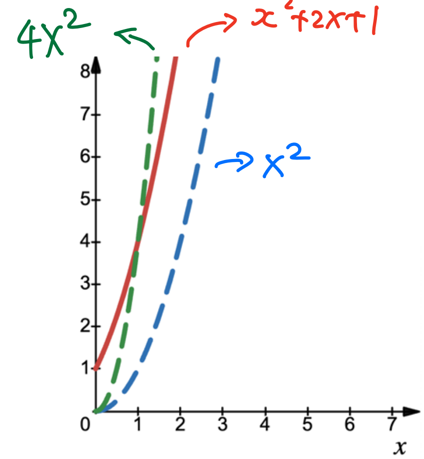
5. Find the big-O for $f(x) = x^2 + 2x + 1$.

$\rightarrow g(x)$

$f(x) = x^2+2x+1 \le 4\boxed{x^2}$, when $x>1$

which means $f(x) = O(x^2)$ with $C=4$, $k=1$

$4x^2$ is an upper bound of x^2+2x+1

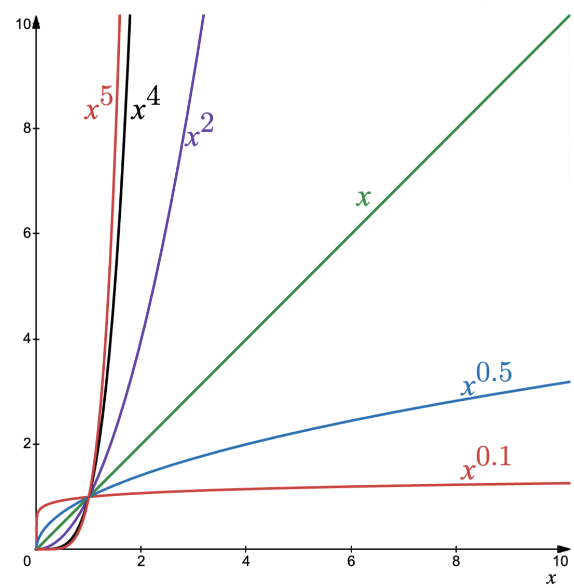


6. The order of growth for functions: Power functions.

$$x^p, p > 0$$

For each $x > 1$, we have

$$\cdots < x^{0.1} \underline{<} x^{0.5} \underline{<} x \underline{<} x^2 \underline{<} x^4 \underline{<} x^5 < \cdots.$$



7. Find the big-O for $f(x) = x^5 + 6x^7 - 2x^2 + 1$.

$f(x) = x^5+6x^7-2x^2+1 \le x^7+6x^7+2x^7+x^7$

$= 10\boxed{x^7}$ for $x>1$

$\Rightarrow f(x)$ is $O(x^7)$ with $C=10$, $k=1$