

MAT2440, Classwork29, Spring2025

ID: _____

Name: _____

1. Algorithms of **Sorting Problems**.

Problem: Given a sequence. Then sort the elements in ascending (or descending) order.

Algorithms: (1) **The Bubble Sort**. (2) **The Insertion Sort**.

For example: input is $\{3, 2, 4, 1, 5\}$ and the output of ascending order is $\{1, 2, 3, 4, 5\}$.

2. Sorting algorithm I: Algorithm and Pseudocode of the **Bubble Sort**.

The smaller elements “bubble” to the top as they are interchanged with larger elements. Here we use an example to explain the algorithm:

Given a list $\{3, 2, 4, 1, 5\}$ and get this list in ascending order $\{1, 2, 3, 4, 5\}$.

compare a_j, a_{j+1}

Algorithm:

	$\hat{j}=1$	$\hat{j}=2$	$\hat{j}=3$	$\hat{j}=4$		$\hat{j}=1$	$\hat{j}=2$	$\hat{j}=3$	
First round $\bar{i}=1$	3 2 5 1 4	2 3 5 1 4	2 3 1 4	2 3 1 4 5	Second round $\bar{i}=2$	2 3 1 4 5	2 3 1 4 5	2 3 1 4 5	2 3 4 5
Third round $\bar{i}=3$	2 1 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	Fourth round $\bar{i}=4$	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5

length of the list

where $i = 1$ to $\underline{5-1}$: how many rounds you have to go through, and

$j = 1$ to $\underline{5-i}$: in each round, how many times you need to check and swap.

Pseudocode:

```

procedure bubblesort( $a_1, a_2, \dots, a_n$ : real numbers with  $n \geq 2$ )
 $n :=$  the length of  $\{a_i\}$ 
for  $i :=$  1 to  $n-1$ 
    for  $j :=$  1 to  $n-i$ 
        if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$ 
    { $a_1, a_2, \dots, a_n$  is in increasing order}
    
```

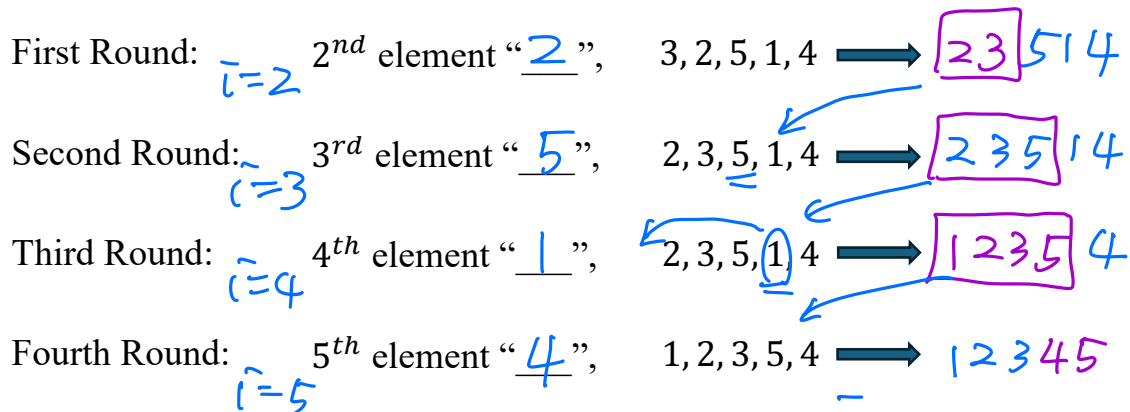
3. Sorting algorithm II: Algorithm and Pseudocode of the **Insertion Sort**.

Insertion sort compares the i^{th} element to its previous $i - 1$ sorted elements and insert the i^{th} element in the right location among the previous $i - 1$ sorted ones.

Here we use the same example to explain the algorithm:

Given a list $\{3, 2, 5, 1, 4\}$ and get this list in ascending order $\{1, 2, 3, 4, 5\}$.

Algorithm:



In " \longrightarrow " parts, we do (1) compare a_i to a_j where $1 \leq j \leq \underline{6}$ until $a_i < a_j$, then (2) insert a_i to the position of a_j .

Pseudocode:

```

procedure insertionsort( $a_1, a_2, \dots, a_n$ : real numbers with  $n \geq 2$ )
 $n :=$  the length of  $\{a_i\}$ 
for  $i :=$  2 to  $n$ 
     $j :=$  1
    while ( $a_i > a_j$  and  $i > j$ )
         $j := j + 1$ 
     $m := a_i$ 
    for  $k :=$  0 to  $i - j - 1$ 
         $a_{i-k} := a_{i-k-1}$ 
     $a_j := m$ 
 $\{a_1, a_2, \dots, a_n$  is in increasing order $\}$ 

```

← pick i^{th} element

} search for index j

} move (a_j, \dots, a_{i-1}) to (a_{j+1}, \dots, a_i)

} insert a_i to the position a_j