MAT2440, Classwork28, Spring2025

ID:_____ Name:_____

1. Searching algorithm II: Algorithm and Pseudocode of the **Binary Search**.

In this algorithm, the list $\{a_i\}$ must be in **ascending** order, that is, $a_1 < a_2 < a_3 < \cdots < a_n$.

Here we use an example to explain the algorithm.

Given a list $\{a_i\}$ with 16 elements where $\{a_i\} =$

$\{a_1,$	$a_2,$	$a_3,$	$a_4,$	$a_5,$	$a_6,$	$a_7,$	$a_8,$	$a_9,$	$a_{10},$	$a_{11},$	$a_{12},$	$a_{13},$	$a_{14},$	$a_{15},$	$a_{16}\}$	$=$
{ 1,	2,	3,	5,	6,	7,	8,	10,	12,	13,	15,	16,	18,	19,	20,	22}	

Algorithm (for searching "$x = 15$" and output the location):

(1) Split this list in half:$\{a_1, \cdots, a_8\}$ and $\{a_9, \cdots, a_{16}\}$, $a_8=10$. $15>10$ (T)

if True$(x > a_8)$ **then** $\{a_9, \cdots, a_{16}\}$
if Fales$(x > a_8)$ **then** $\{a_1, \cdots, a_8\}$

(2) Split the list in half: $\{a_9, \cdots, a_{12}\}$ and $\{a_{13}, \cdots, a_{16}\}$, $a_{12}=16$ $15>16$ (F)

if True$(x > a_{12})$ **then** $\{a_{13}, \cdots, a_{16}\}$
if Fales$(x > a_{12})$ **then** $\{a_9, \cdots, a_{12}\}$

(3) Split the list in half: $\{a_9, a_{10}\}$ and $\{a_{11}, a_{12}\}$, $a_{10}=13$ $15>13$ (T)

if True$(x > a_{10})$ **then** $\{a_{11}, a_{12}\}$
if Fales$(x > a_{10})$ **then** $\{a_9, a_{10}\}$

(4) Split the list in half: $\{a_{11}\}$ and $\{a_{12}\}$, $a_{11}=15$. $15>15$ (F)

if True$(x > a_{11})$ **then** $\{a_{12}\}$
if Fales$(x > a_{11})$ **then** $\{a_{11}\}$

(5) Comparing x and a_{11},

if True $(x = a_{11})$ **then** location= 11
if False $(x = a_{11})$ **then** location= 0

15 is NOT in the list

Pseudocode:

procedure *binary_search*(x: integer, $a_1, a_2, \cdots, a_n$: distinct integers)
$n :=$ the length of $\{a_i\}$
$i :=$ 1 (which is left end location)
$j :=$ n (which is right end location)
while $(i \le j)$
    m:= $\left\lfloor \frac{i+j}{2} \right\rfloor$ (which is the locate the middle of the sequence)
    if $x > a_m$ **then** $i :=$ m+1
        else $j :=$ m
if $x =$ a_i **then** *location* := i
    else *location* := 0
return location { *location* is the subscript of the term that equals x, or 0 if x is not found.}

6. How does this pseudocode work with $\{1, 2, 3, 5, 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 20, 22\}$ and searching for $x = 15$?

How many elements do we have

Initialization: $n =$ 16, $i =$ 1, and $j =$ 16.

* : do nothing.

round	$i < j$ (T/F)	m	a_m	$x > a_m$ (T/F)	$i = m + 1$	$j = m$
1	1<16 (T)	$\lfloor \frac{1+16}{2} \rfloor = 8$	10	15>10 (T)	i=9	* (j=16)
2	9<16 (T)	$\lfloor \frac{9+16}{2} \rfloor = 12$	16	15>16 (F)	* (i=9)	j=12
3	9<12 (T)	$\lfloor \frac{9+12}{2} \rfloor = 10$	13	15>13 (T)	i=11	* (j=12)
4	11<12 (T)	$\lfloor \frac{11+12}{2} \rfloor = 11$	15	15>15 (F)	* (i=11)	j=11
5	11<11 (F)	stop				

i=1, j=16
i=9, j=16
i=9, j=12
i=11, j=12
i=11, j=11

($x = a_i$) is true implies *location* = 11.

15 = 15

Return: location = 11

which is "i"

7. How does this pseudocode work with $\{2, 3, 8, 9\}$ and searching for $x = 6$?

a_1 a_2 a_3 a_4

Initialization: $n =$ 4, $i =$ 1, and $j =$ 4.

T F

round	$i < j$ (T/F)	m	a_m	$x > a_m$ (T/F)	$i = m + 1$	$j = m$
1	1<4 (T)	$\lfloor \frac{1+4}{2} \rfloor = 2$	3	6>3 (T)	i=3	X (j=4)
2	3<4 (T)	$\lfloor \frac{3+4}{2} \rfloor = 3$	8	6>8 (F)	X (i=3)	j=3
3	3<3 (F)	stop				

i=1, j=4
i=3, j=4
i=3, j=3

($x = a_i$) is F implies *location* = 0.

6 = a_3=8 (F)

Return: location = 0 ← 6 is NOT in the list.