MAT2440, Classwork24, Spring2025

ID:________________________ Name:________________________

1. The special Sequence with explicit formula: **Arithmetic Sequences**

An arithmetic sequence $\{a_n\}$ is a sequence of the form $\boldsymbol{a_n = a_1 + (n-1)d}$:

$$a_1 = a_1,\ a_2 = a_1 + d,\ a_3 = a_1 + 2d, \cdots, a_k = a_1 + (k-1)\cdot d, \cdots,$$

where the initial (first) term a_1 and the common difference d are real numbers.

2. List the first five terms $a_1, a_2, \cdots, a_5$ of the arithmetic sequence $\{a_n\}$ and find the common difference d of the sequence. (a) $a_n = 3 + (n-1)(-4)$. (b) $a_n = -1 + 4n$.

(a)
$a_1 = 3 + (1-1)\cdot(-4) = 3$ -4
$a_2 = 3 + (2-1)\cdot(-4) = -1$ -4
$a_3 = 3 + (3-1)\cdot(-4) = -5$ -4
$a_4 = 3 + (4-1)\cdot(-4) = -9$ -4
$a_5 = 3 + (5-1)(-4) = -13$

$d = -4$

(b)
$a_1 = -1 + 4\cdot 1 = 3$ $+4$
$a_2 = -1 + 4\cdot 2 = 7$ $+4$
$a_3 = -1 + 4\cdot 3 = 11$ $+4$
$a_4 = -1 + 4\cdot 4 = 15$ $+4$
$a_5 = -1 + 4\cdot 5 = 19$

$d = 4$

3. . The special Sequence with explicit formula: **Geometric Sequences**

An geometric sequence $\{a_n\}$ is a sequence of the form $a_n = a_1 r^{n-1}$:

$$a_1 = a_1,\ a_2 = a_1 r,\ a_3 = a_1 r^2, \cdots, a_k = a_1 r^{k-1}, \cdots,$$

where the initial term a_1 and the common ratio. r are real numbers.

4. List the first five terms $a_1, a_2, \cdots, a_5$ of the geometric sequence $\{a_n\}$ and find the common ratio r of the sequence. (a) $a_n = (-1)^n$. (b) $a_n = \left(-\frac{1}{2}\right)^{n-1}$.

(a)
$a_1 = (-1)^1 = -1$
$a_2 = (-1)^2 = 1$
$a_3 = (-1)^3 = -1$
$a_4 = (-1)^4 = 1$
$a_5 = (-1)^5 = -1$

$r = \frac{a_2}{a_1} = \frac{1}{-1} = -1$

(b)
$a_1 = \left(-\frac{1}{2}\right)^{1-1} = \left(-\frac{1}{2}\right)^0 = 1$
$a_2 = \left(-\frac{1}{2}\right)^{2-1} = -\frac{1}{2}$
$a_3 = \left(-\frac{1}{2}\right)^{3-1} = \frac{1}{4}$
$a_4 = \left(-\frac{1}{2}\right)^{4-1} = -\frac{1}{8}$
$a_5 = \left(-\frac{1}{2}\right)^{5-1} = \frac{1}{16}$

$r = \frac{a_2}{a_1} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}$

5. Define a Sequence by **Recursive Relations**:

Another popular method to define a sequence is to provide one or more initial terms together with a recursive rule for determining subsequent terms from those that precede them.

6. Let $\{a_n\}$ be a sequence that satisfies the **initial term** $a_0 = 2$ and the **recurrence relation**

$$a_n = a_{n-1} + 3 \text{ for } n = 1, 2, 3, \cdots. \Rightarrow a_n - a_{n-1} = 3$$

What are a_1, a_2, and a_3? $a_0 = 2 \leftarrow$ initial term

$a_1 = a_{1-1} + 3 = a_0 + 3 = 2+3 = 5$

$a_2 = a_{2-1} + 3 = a_1 + 3 = 5+3 = 8$

$a_3 = a_{3-1} + 3 = a_2 + 3 = 8+3 = 11$

this is an arithmetic sequence with $d = 3$

$(a_n = a_0 + n \cdot 3)$

7. Let $\{a_n\}$ be a sequence that satisfies the **initial term** $\boxed{a_0 = 3}$ and the **recurrence relation**

$$a_n = \frac{1}{3} a_{n-1} \text{ for } n = 1, 2, 3, \cdots. \Rightarrow \frac{a_n}{a_{n-1}} = \frac{1}{3}$$

What are a_1, a_2, and a_3? $a_0 = 3$

$a_1 = \frac{1}{3} a_{1-1} = \frac{1}{3} a_0 = \frac{1}{3} \cdot 3 = 1$

$a_2 = \frac{1}{3} a_{2-1} = \frac{1}{3} a_1 = \frac{1}{3} \cdot 1 = \frac{1}{3}$

$a_3 = \frac{1}{3} a_{3-1} = \frac{1}{3} a_2 = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

This is a geometric sequence with common ratio $r = \frac{1}{3}$

$\left(a_n = a_0 \left(\frac{1}{3}\right)^n = 3 \cdot \left(\frac{1}{3}\right)^n\right)$

8. (*Fibonacci sequence*) Let $\{f_n\}$ be a sequence that satisfies the **initial term** $f_0 = 1$, $f_1 = 1$, and **recurrence relation**

$$f_n = f_{n-1} + f_{n-2} \text{ for } n = 2, 3, 4, \cdots.$$

What are the first five terms?

$f_0 = 1$ ← initial terms

$f_1 = 1$

$f_2 = f_{2-1} + f_{2-2} = f_1 + f_0 = 1+1 = 2$

$f_3 = f_{2} + f_{1} = 2+1 = 3$ ($2 \to 3-1$, $1 \to 3-2$)

$f_4 = f_3 + f_2 = 3+2 = 5$

$\{1, 1, 2, 3, 5, 8, 13, 21, \cdots\}$

Explicit formula (also called a closed formula) of *Fibonacci sequence*:

$$f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right], \quad n = 0, 1, 2, \cdots$$