MAT2440, Classwork23, Spring2025

ID:______________________________ Name:______________________________

1. Let $f: \mathbb{Z} \to \mathbb{Z}$ be such that $f(x) = x + 1$. Is f invertible, and if it is, what is its inverse?

Is f invertible? we have to show $f(x)$ is one-to-one and onto

① To show f is one-to-one

Let $f(a)=f(b) \Rightarrow a+1=b+1$

$\Rightarrow a=b$, Yes, $f(x)$ is one-to-one

② To show f is onto. Let $y=f(x)$, $y=x+1$

$\forall y \in \mathbb{Z}$, we have $x+1=y \Rightarrow x=y-1 \in \mathbb{Z}$

for all output $y\in\mathbb{Z}$, there is an input $x\in\mathbb{Z}$

Find f^{-1}

① Let $y=f(x)$, we have $y=x+1$

② switch x & y: $x=y+1$

③ solve for y: $y+1=x$

$\Rightarrow y=x-1$

④ Replace y by f^{-1} $\Rightarrow f^{-1}(x)=x-1$.

2. Let $f: \mathbb{R} \to \mathbb{R}$ be such that $f(x) = x^2$. Is f invertible?

Let $a=1$, $b=-1$,

$f(a)=(1)^2=1$,

$f(b)=(-1)^2=1$

for different inputs, we get the same output which implies f is NOT one-to-one

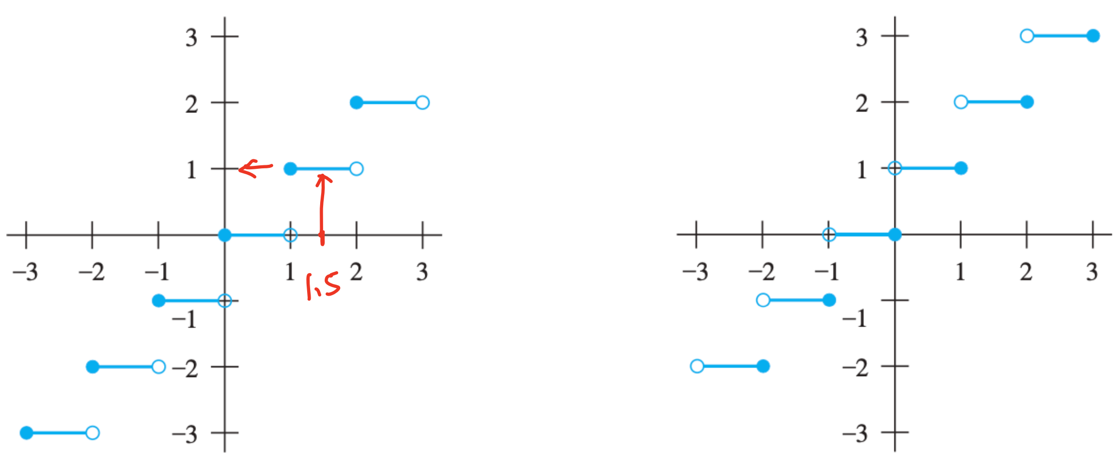
$\Rightarrow f$ is NOT bijection

$\Rightarrow f$ is NOT invertible

3. The definition of the **Floor** function and the **Ceiling** function:

(a) The __floor__ function, denoted by __$\lfloor x \rfloor$__, assigns to real number x the __largest__ integer that is __less than__ or __equal to__ x. That is, $\lfloor x \rfloor$ = **largest integer** $\le x$.

$\lfloor 1.5 \rfloor$ = __1__. $\lfloor 0.2 \rfloor$ = __0__. $\lfloor 2 \rfloor$ = __2__. $\lfloor -1.5 \rfloor$ = __−2__. $\lfloor -3 \rfloor$ = __−3__.



(a) $y = \lfloor x \rfloor$

(b) $y = \lceil x \rceil$

(b) The __Ceiling__ function, denoted by __$\lceil x \rceil$__, assigns to real number x the __smallest__ integer that is __more than__ or __equal to__ x. That is, $\lceil x \rceil$ = **smallest integer** $\ge x$.

$\lceil 1.5 \rceil$ = __2__. $\lceil 0.2 \rceil$ = __1__. $\lceil 2 \rceil$ = __2__. $\lceil -1.5 \rceil$ = __−1__. $\lceil -3 \rceil$ = __−3__.

1. The introduction of a Sequence:

A <u>sequence</u> is a discrete structure to represent an <u>ordered</u> list. For example, $1, 2, 4, 8, 16$ is a sequence with <u>5</u> terms and $1, 2, 4, 8, 16, \cdots, 2^n, \cdots$ is an <u>infinite</u> one.

2. Differences between sequences and sets:

The <u>order</u> matters in sequences: $\boxed{\text{sequence}}$ $\overset{a_1}{1}, \overset{a_2}{3}, \overset{a_3}{5}, \overset{a_4}{7}, \overset{a_5}{9}$ <u>$\neq$</u> sequence $5, 3, 1, 7, 9$, but

$\boxed{\text{set}}$ $\{1, 3, 5, 7, 9\}$ <u>$=$</u> set $\{5, 3, 1, 7, 9\}$.

The meaning of repeated number: $2, 2, 2, 2$ is a sequence with four terms but set $\{2, 2, 2, 2\}$ is essentially <u>$\{2\}$</u>.

3. The definition of a Sequence:

A sequence is a <u>function</u> from the input $\{1, 2, \cdots, n, \cdots\}$ to the output $\{a_1, a_2, \cdots, a_n, \cdots\}$ which a_n represents the n^{th} <u>term</u> of the sequence and the sequence is denoted by <u>$\{a_n\}$</u>

- Explicit Formula or Recursive Relation (Formula)

4. Define a Sequence by **Explicit Formula** $a_n = f(n)$:

When a sequence is defined in an <u>explicit</u> formula $f(n)$, the value of the i^{th} term a_i in this sequence can by computed by $f(i)$.

5. Consider the sequence $\{a_n\}$, where $a_n = \frac{1}{n}$. Then list the first four terms of the sequence, beginning with a_1.

$a_1 = \frac{1}{1} = 1$

$a_2 = \frac{1}{2}$

$a_3 = \frac{1}{3}$

$a_4 = \frac{1}{4}$

6. List the first five terms $a_0, a_1, \cdots, a_4$ of the sequence $\{a_n\}$, where $a_n = \left\lceil \frac{n}{2} \right\rceil$.

$a_0 = \left\lceil \frac{0}{2} \right\rceil = \lceil 0 \rceil = 0$

$a_1 = \left\lceil \frac{1}{2} \right\rceil = 1$

$a_2 = \left\lceil \frac{2}{2} \right\rceil = \lceil 1 \rceil = 1$

$a_3 = \left\lceil \frac{3}{2} \right\rceil = \lceil 1.5 \rceil = 2$

$a_4 = \left\lceil \frac{4}{2} \right\rceil = \lceil 2 \rceil = 2$