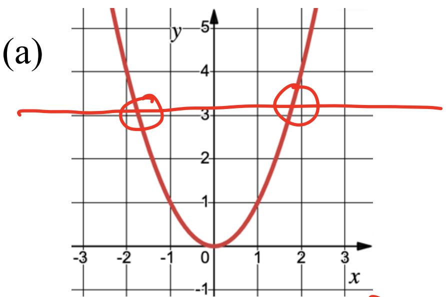
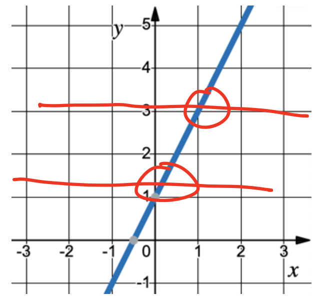
MAT2440, Classwork22, Spring2025

ID:______________________________ Name:______________________________

1. Use **Horizontal Line Test** to determine which of the following are the graphs of one-to-one functions. (a)



(b)



NOT a one-to-one function
(more than one intersection per horizontal line)

Yes, it is one-to-one
(at most one intersection point per horizontal line)

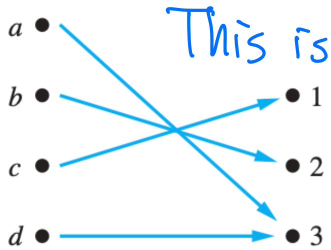
2. The definition of an **Onto** Function:

A function f from A to B is called Onto, or a subjection, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$ and we say f is **subjective**. Hence,

$$\forall b \in B \exists a \in A(f(a) = b).$$

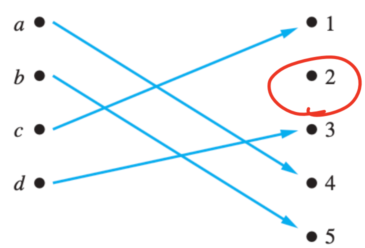
3. Check if the given function (or mapping) an onto function.

(a)



This is Onto

(b)



This is NOT onto

The output "2" does not have an input.

4. Check if the given function an onto function. (a) $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$.

(b) $g(x) = 2x + 1$ from $\mathbb{R}$ to $\mathbb{R}$. (c) $h(x) = 2x + 1$ from $\mathbb{Z}$ to $\mathbb{Z}$.

To prove f is NOT Onto: provide a counterexample to show ~~that~~ there exists an output without input.

(a) NO. here is an counterexample: Let $b = -1 \in \mathbb{R}$, we have

$x^2 = -1$ which has no solution in $\mathbb{R}$.

$\Rightarrow$ NOT all output has an input $\Rightarrow$ $f(x) = x^2$ from $\mathbb{R}$ to $\mathbb{R}$ is NOT onto.

(c) NO, here is an counterexample, Let $b = 2 \in \mathbb{Z}$, we have

$2x + 1 = 2 \Rightarrow x = \frac{1}{2} \notin \mathbb{Z}$

$\Rightarrow$ NOT all output has an input $\Rightarrow$ $h(x) = 2x+1$ from $\mathbb{Z}$ to $\mathbb{Z}$ is NOT onto.

To prove f is onto: need to show that $\forall y \in B$, we can solve for $x \in A$, such that $f(x) = y$.

(b) To show "$g(x) = 2x+1$ from $\mathbb{R}$ to $\mathbb{R}$". is onto, we have

Given $y \in \mathbb{R}$, $2x+1 = y$

$\Rightarrow 2x = y-1$

$\Rightarrow x = \frac{y-1}{2}$

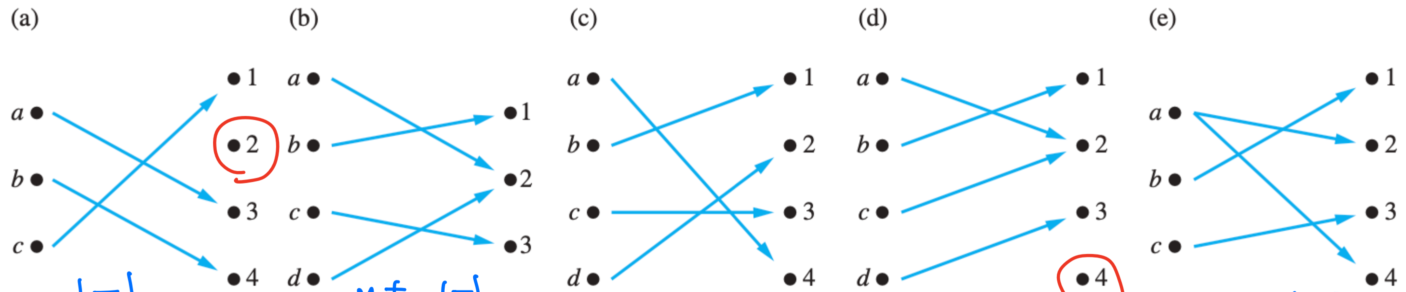
$\forall y \in \mathbb{R}$, we can find $x = \frac{y-1}{2}$ which is also a real number.

Therefore, $g(x) = 2x+1$ from $\mathbb{R}$ to $\mathbb{R}$ is onto.

5. The definition of a **Bijective** Function:

If a function f is both one-to-one and onto, then f is bijective.

6. Check if the given function (or mapping) a bijective function.



(a) 1-1 but not onto

(b) not 1-1 but it is onto

(c) both, or bijection

(d) neither (but still a function)

(e) NOT a function

7. Show that $f(x) = x^3$ from $\mathbb{R}$ to $\mathbb{R}$ is bijective.

(1) To show f(x) is one to one

Let $f(a) = f(b)$, then $a^3 = b^3 \Rightarrow \sqrt[3]{a^3} = \sqrt[3]{b^3} \Rightarrow a = b$

Yes f(x) is one-to-one

(2) To show f(x) is onto

Let $f(x) = y$, $y = x^3$.

$\forall y \in \mathbb{R}$, we have $x^3 = y$. $\xrightarrow{\text{solve for } x}$ $\sqrt[3]{x^3} = \sqrt[3]{y}$

$\Rightarrow x = \sqrt[3]{y} \in \mathbb{R}$

(for all output y, we can find an input $x = \sqrt[3]{y} \in \mathbb{R}$)

Yes, f(x) is onto. Therefore, by (1) & (2), f(x) is bijection.

8. The definition of an **Inverse** Function:

Let a function f be one-to-one from a set A to a set B. Then f is invertible and the inverse function of f, denoted f^{-1}, assigns $f^{-1}(y) = x$ if $f(x) = y$.