MAT2440, Classwork21, Spring2025

ID:_____ Name:_____

1. The definition of a Function:

Let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A, denoted by $f: A \xrightarrow{\text{maps}} B$ which is read as f maps A to B. Then we say that A is the domain of f and B is the codomain of f.

2. If $f(a) = b$, we say that b is the image of a and the range of f is the set of all images of elements of A.

range $\subseteq$ codomain

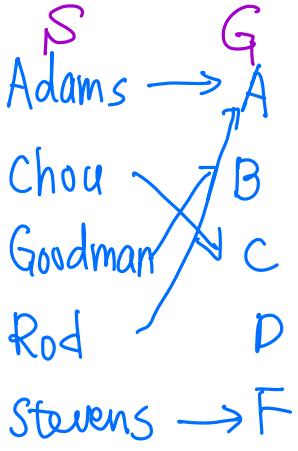
What is the difference between range and codomain

3. Suppose that each student in 2440 is assigned a letter grade from the set $G = \{A, B, C, D, F\}$. And suppose that grades are A for Adams, C for Chou, B for Goodman, A for Rod, and F for Stevens. Let $S = \{\text{Adams, Chou, Goodman, Rod, Stevens}\}$ be the set of the students.

(a) Is $f: S \to G$ a function? (b) What is the domain, range, and codomain of f?

(c) Is $g: G \to S$ a function?

(a)



Yes, f is a function

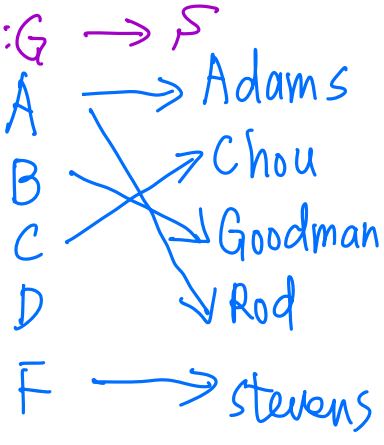
(b)

domain: = S

range: = {A, B, C, F}

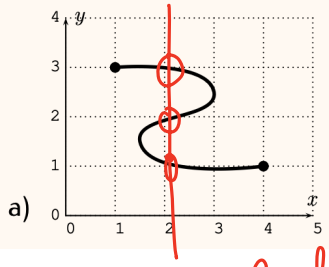
codomain: = {A, B, C, D, F}

(c) $g: G \to S$

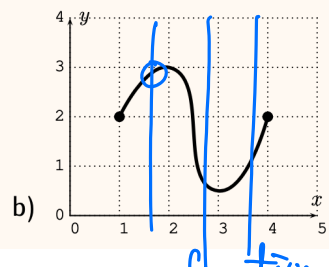


NO, g is NOT a function

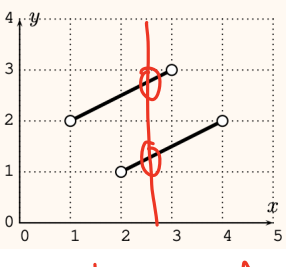
4. Use **Vertical Line Test** to determine which of the following are the graphs of functions.

a) 

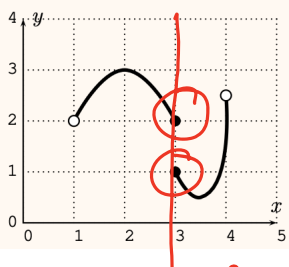
NOT a function

b) 

a function (at most one intersection point per vertical line)

c) 

NOT a function

d) 

NOT a function.

5. The definition of a One-to-One Function:

A function f is said to be 1-1 (1-to-1) One-to-One, or an injection, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f and we say f is **injective**. Hence, we have

$\forall a \forall b (f(a) = f(b) \rightarrow$ a=b) or $\forall a \forall b (a \neq b \rightarrow$ f(a)≠f(b))

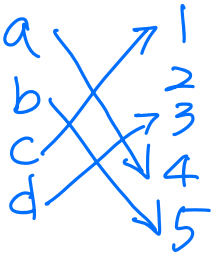
One-to-one function means different inputs will produce different outputs.

6. Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5\}$.

(a) Let $f: A \rightarrow B$ with $f(a) = 4, f(b) = 5, f(c) = 1$, and $f(d) = 3$. Is f one-to-one?

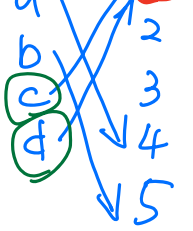
(b) Let $g: A \rightarrow B$ with $g(a) = 4, g(b) = 5, g(c) = 1$, and $g(d) = 1$. Is g one-to-one?

(a) $f: A \rightarrow B$



Yes, f is one-to-one

(b) $g: A \rightarrow B$



No; g is NOT one-to-one (an output has two inputs)

7. Is $f(x) = x^2$ a one-to-one function?

Is $a \neq b \rightarrow f(a) \neq f(b)$ for all real number?

Sol: Here is a counterexample when $a=1$, $b=-1$ $(a \neq b)$ $(a=2, b=-2)$

$f(a) = f(1) = 1^2 = 1$

$f(b) = f(-1) = (-1)^2 = 1$

$\Rightarrow f(a) = f(b)$

Therefore, it implies that $f(x)=x^2$ is NOT one-to-one.

- To prove f is NOT one-to-one:
  need to provide a counterexample to show
  $a \neq b$, but $f(a) = f(b)$
- To prove f is one-to-one:
  need to show $\forall a \, \forall b \, (f(a) = f(b) \rightarrow a = b)$

For example, to show $f(x) = 2x+1$ is one to one, we assume $f(a) = f(b)$ which implies $2a+1 = 2b+1$ for arbitrary a, b in the domain of f.

Then, we have $2a+1 = 2b+1$

$\Rightarrow 2a = 2b$

$\Rightarrow a = b$ for arbitrary a, b in domain of f.

Therefore, since a, b are arbitrary and $f(a) = f(b)$ implies $a = b$,
then $f(x) = 2x+1$ is one-to-one