MAT2440, Classwork20, Spring2025

ID:__________________________________ Name:_____________________________________

1. Use the membership table to prove one of the De Morgan's law: $\overline{A \cap B} = \bar{A} \cup \bar{B}$.

	A	B	$A \cap B$	$A \cup B$	$\bar{A}$	$\bar{B}$	$\bar{A} \cup \bar{B}$	$\overline{A \cap B}$
$a \in A$ $a \in B$	1	1	1	1	0	0	0	0
$a \in A$ $a \notin B$	1	0	0	1	0	1	1	1
$a \notin A$ $a \in B$	0	1	0	1	1	0	1	1
$a \notin A$ $a \notin B$	0	0	0	0	1	1	1	1

2. The Computer Representation of Sets:

Let $\boldsymbol{S}$ be a set and $\boldsymbol{U}$ be the universal set. If the universal set U is finite, then S can be represented bit strings.

3. Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the ordering of element of U has the elements in increasing order.

(a) What bit strings can represent the subset of all odd integers in U?

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\{1, 3, 5, 7, 9\}$ odd integers in U

string 1 0 1 0 1 0 1 0 1 0

(b) What bit strings can represent the set $B = \{1, 2, 5, 6\}$ in U?

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$B = \{1, 2, 5, 6\}$

1 1 0 0 1 1 0 0 0 0 ← string of B.

(c) What set in U can be represented by the bit string 00 1111 0010?

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

0 0 1 1 1 1 0 0 1 0

$\Rightarrow \{3, 4, 5, 6, 9\}$

(d) Let $A_1 = \{1, 2, 3, 4, 5\}$ and $A_2 = \{1, 3, 5, 7, 9\}$ in U. Use bit string to find $A_1 \cup A_2$ and $A_1 \cap A_2$.

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A_1 = \{1, 2, 3, 4, 5, \}$

1 1 1 1 1 0 0 0 0 0 ← string of A_1

1 0 1 0 1 0 1 0 1 0 ← string of A_2

string of $A_1 \cup A_2$: 1 1 1 1 1 0 1 0 1 0 ⇒ $\{1, 2, 3, 4, 5, 7, 9\}$

string of $A_1 \cap A_2$: 1 0 1 0 1 0 0 0 0 0 ⇒ $\{1, 3, 5\}$

4. The Generalized Unions and Intersections with the **finite** family of sets:

The union of the sets $A_1, A_2, \cdots, A_n$: $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$ = $\bigcup_{i=1}^{n} A_i$.

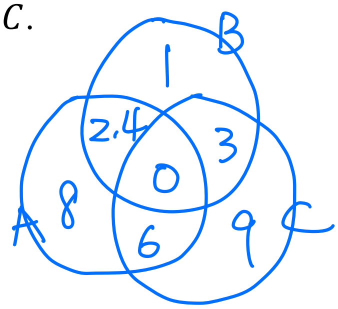
The intersection of the sets $A_1, A_2, \cdots, A_n$: $A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$ = $\bigcap_{i=1}^{n} A_i$.

5. The Generalized Unions and Intersections with the **infinite** family of sets:

The union of the sets $A_1, A_2, \cdots, A_n, \cdots$: $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n \cup \cdots$ = $\bigcup_{i=1}^{\infty} A_i$.

The intersection of the sets $A_1, A_2, \cdots, A_n, \cdots$: $A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n \cdots$ = $\bigcap_{i=1}^{\infty} A_i$.

6. Let $A = \{0, 2, 4, 6, 8\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{0, 3, 6, 9\}$. Then find (a) $A \cup B \cup C$ and (b) $A \cap B \cap C$.



(a) $A \cup B \cup C = \{0, 1, 2, 3, 4, 6, 8, 9\}$
(all the elements)

(b) $A \cap B \cap C = \{0\}$
(the common one through all sets)

7. Suppose that $A_i = \{1, 2, 3, \cdots, i\}$ for $i = 1, 2, 3, \cdots$. Then find (a) $\bigcup_{i=1}^{\infty} A_i$ and (b) $\bigcap_{i=1}^{\infty} A_i$

$A_1 = \{1\}$

$A_2 = \{1, 2\}$

$A_5 = \{1, 2, 3, 4, 5\}$

$A_{100} = \{1, 2, 3, 4, 5, \cdots, 98, 99, 100\}$

(a) $\bigcup_{i=1}^{\infty} A_i = \{1, 2, 3, 4, \cdots, n, \cdots\}$

(b) $\bigcap_{i=1}^{\infty} A_i = \{1\}$.