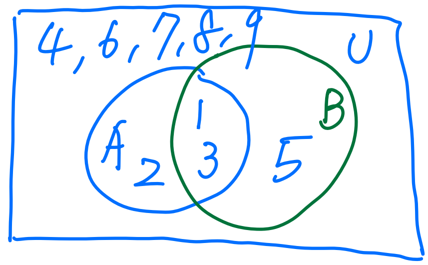
MAT2440, Classwork19, Spring2025

ID:______________________________ Name:______________________________

1. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$, and $U = \{x | x \in \mathbb{Z}^{+} \wedge x < 10\}$ $= \{1,2,3,4,5,6,7,8,9\}$. Find the following sets:



(a) $A \cup B = \{1, 2, 3, 5\}$

(b) $A \cap B = \{1, 3\}$

(c) $A - B = \{2\}$

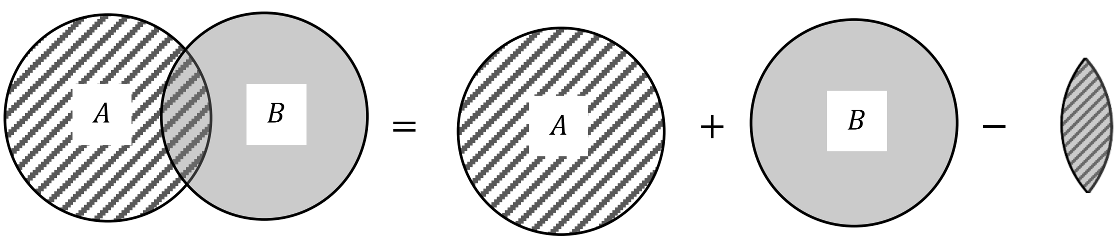
(d) $B - A = \{5\}$

(e) $\bar{A} = \{4, 5, 6, 7, 8, 9\}$

(f) $\bar{B} = \{2, 4, 6, 7, 8, 9\}$

2. The **Principle of Inclusion-Exclusion** for two sets:

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be sets. We have $|A \cup B| = |A| + |B| - |A \cap B|$



3. Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$. Show that $|A \cup B| = |A| + |B| - |A \cap B|$.

$|A \cup B| = 4$, $|A \cap B| = 2$, $|A| = 3$, $|B| = 3$

$|A \cup B| = |A| + |B| - |A \cap B|$

$4 = 3 + 3 - 2$

4. The Principle of Inclusion-Exclusion for **three** sets:

Let $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ be sets. Then we have

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|.$$

5. Table of Set Identities: Let $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ be sets and $\boldsymbol{U}$ be the universal set.

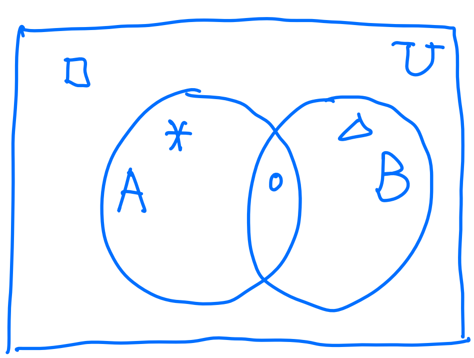
Name	**Identity**
Identity laws	$A \cap U = A$ $A \cup \emptyset = A$
Domination laws	$A \cup U = U$ $A \cap \emptyset = \phi$
Idempotent laws	$A \cap A = A$ $A \cup A = A$
Complementation laws	$\overline{(\bar{A})} = A$
Commutative laws	$A \cup B = B \cup A$ $A \cap B = B \cap A$
Associative laws	$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's laws	$\overline{A \cup B} = \bar{A} \cap \bar{B}$ $\overline{A \cap B} = \bar{A} \cup \bar{B}$
Absorption laws	$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$
Complement laws	$A \cup \bar{A} = U$ $A \cap \bar{A} = \phi$

6. The Membership tables:

To prove the set identities, similar as Truth table in Logic operations, we have membership tables: In membership tables, let S be a set and we have

If $a \in S$, we assign 1 (like the T (true) in truth table)

If $a \notin S$, we assign 0 (like the F (false) in truth table)



$*$ $a \in A$ $a \notin B$

$\triangle$ $a \notin A$ $a \in B$

$\circ$ $a \in A$ $a \in B$

$\square$ $a \notin A$ $a \notin B$