MAT2440, Classwork17, Spring2025

ID:______________________ Name:______________________

1. The Intervals: sets of all real numbers between two numbers

   Let a and b be real numbers with $a \leq b$. We have

   $[a, b] = \{x \mid a \leq x \leq b\}$, $[a, b) = \{x \mid a \leq x < b\}$,
   $(a, b] = \{x \mid a < x \leq b\}$, $(a, b) = \{x \mid a < x < b\}$.

2. The definition of Subsets:

   $B \supseteq A$

   Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two sets. We say $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$, and $\boldsymbol{B}$ is a superset of $\boldsymbol{A}$, if and only if every element of $\boldsymbol{A}$ is also an element of $\boldsymbol{B}$. We use the notation $A \subseteq B$ to indicate $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$:

   $$\forall x(x \in A \rightarrow x \in B) \equiv A \subseteq B$$

   belongs to

3. Showing that $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$ ($\boldsymbol{A} \subseteq \boldsymbol{B}$): If x belongs to $\boldsymbol{A}$, then x also belongs to B.

   Example: $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$.

4. Showing that $\boldsymbol{A}$ is **Not** a subset of $\boldsymbol{B}$ ($\boldsymbol{A} \nsubseteq \boldsymbol{B}$): Find a single $x \in \boldsymbol{A}$ such that $x \notin B$.

   Example: Let $A = \{0, 1, 2\}, B = \{1, 2, 3, 4\}$, then $A \nsubseteq B$ because $0 \in A$ but $0 \notin B$

5. For any set S, we have

   $\emptyset \subseteq S$: Empty set is a subset of S.
   $S \subseteq S$: S is a subset of S itself.

6. A Proper Subset:

   Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two sets. We say $\boldsymbol{A}$ is a proper subset of $\boldsymbol{B}$ if $\boldsymbol{A}$ is a subset of $\boldsymbol{B}$ but $A \neq B$, and it is denoted by $A \subset B$.

   Example: Let $A = \{1, 2\}, B = \{1, 2, 3, 4\}$. $A \subset B$

$B \not\subset B$ $B \subseteq B$ $A \subseteq B$

7. Showing Two sets are Equal: Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be two sets.

To show $\boldsymbol{A}$ and $\boldsymbol{B}$ are **equal**, we need to prove that $A \subseteq B$ and $B \subseteq A$.

$A \subseteq B$ ↑ for set $a \in A$ ↖ for element.

8. Let $S = \{\emptyset, \{1\}, \{1,2\}\}$, $B = \{1, \{1\}\}$, and $C = \{\emptyset, \{1\}\}$. Complete the following relations:

(a) $C \subseteq S$. (b) $B \not\subseteq S$. (c) $\{1\} \in S$. (d) $\emptyset \subseteq S$. (e) $1 \in B$.

$1 \in B$, $1 \notin S$ $\{1\} \in S$

$\{1\} \not\subseteq S$ $\{\{1\}\} \subseteq S$

$\in$

9. The definition of **Power Set**:

Given a set S, the power set of S is the set of all subsets of S and it is denoted by $P(S)$.

If a set has n elements, then its power set has 2^n elements.

10. What is the power set of the set $\{0,1,2\}$ (denoted by $\mathcal{P}(\{0,1,2\})$)?

$$P(\{0,1,2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{1,2\}, \{0,2\}, \{0,1,2\}\}$$

11. The definition of **Cartesian Products**:

Let $\boldsymbol{A}$ and $\boldsymbol{B}$ be sets. The Cartesian products of $\boldsymbol{A}$ and $\boldsymbol{B}$, denoted by $A \times B$, is

The set of all **ordered pairs** (a, b), where $a \in A$ and $b \in B$. Hence,

$$A \times B = \{(a, b) \mid a \in A \wedge b \in B\}.$$

and

12. Let $\boldsymbol{A} = \{1, 2\}$ and $\boldsymbol{B} = \{a, b, c\}$. Find the Cartesian Products $A \times B$ and $B \times A$.

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

$$B \times A = \{(a,1), (b,1), (c,1), (a,2), (b,2), (c,2)\}$$

$$A \times B \neq B \times A$$