MAT2440, Classwork14, Spring2025

ID:______________________________ Name:______________________________

1. **Universal instantiation**: It is the rule of inference that given the premise $\forall x P(x)$, and it concludes that $P(c)$ is __true__, where c is a particular member of the domain.

2. **Universal generalization**: It is the rule of inference that given the premise $P(c)$ is true for all elements c in the domain, and it concludes that $\forall x P(x)$ is __true__.

3. **Existential instantiation**: It is the rule of inference that if we know $\exists x P(x)$ is true, it concludes that there is an element c in the domain for which $P(c)$ is __true__.

4. **Existential generalization**: It is the rule of inference that when a particular element c with $P(c)$ true is known, it concludes that $\exists x P(x)$ is __true__.

5. The Rules of Inference for Quantified Statements:

Name	**Rule of Inference**
Universal instantiation	$\forall x P(x)$ $\therefore P(c)$, c is in the domain of x
Universal generalization	$P(c)$ for an arbitrary c $\therefore \forall x P(x)$, x includes all c
Existential instantiation	$\exists x P(x)$ $\therefore P(c)$, for some c in the domain of x
Existential generalization	$P(c)$ for some element c $\therefore \exists x P(x)$, x includes a element c.

6. Show that the premises ''Everyone in Mat2440 has taken a course in CS'' and ''Marla is a student in Mat2440'' implies the conclusion ''Marla has taken a course in CS.''

$S(x)$: x is in Mat2440 , $C(x)$: x has taken a CS course

$\forall x\,(S(x) \to C(x))$ premise #1 ② we can get premise #3 by premise #1.

$S(\text{Marla}) \to C(\text{Marla})$ premise #3 ① We need this premise to get the conclusion

$S(\text{Marla})$ premise #2

$\therefore C(\text{Marla})$ conclusion we want

1. Introduction of Proofs:

A proof is a valid argument that establishes the truth of a mathematical statement.

A theorem is a statement that can be shown to be true. We demonstrate that a theorem is true with a **proof**. The statement used in a proof can include axioms, the premises of the theorem, and previously proven theorems.

2. The First Method: A Direct Proof

A direct proof shows that a conditional statement $p \rightarrow q$ is true by showing that if p is true, then q must also be true. The direct proofs are quite straightforward, but sometimes require **particular insights** and can be quite **tricky**.

3. Even or Odd Integers:

The integer n is ***even*** if there exists an integer k such that $n =$ $2 \cdot k$.

The integer n is ***odd*** if there exists an integer k such that $n =$ $2k+1$.

4. Give a direct proof of the theorem ‘‘If n is an odd integer, then n^2 is odd.’’

Proof: $P(n)$ is "n is odd", $Q(n)$: n^2 is odd.

$\forall n\ (P(n) \rightarrow Q(n))$

(n is an odd integer)

Let $n = 2k+1$ (assume n is odd)

then $n^2 = (2k+1)(2k+1)$

$= 4k^2+2k+2k+1$

$= 2(2k^2+k+k)+1$

which means n^2 is odd.

5. Give a direct proof of the theorem ‘‘If n^2 is an odd integer, then n is odd.’’

$Q(n) \rightarrow P(n)$

Let $n^2 = 2k+1$ then $n = \sqrt{2k+1}$.