

# MAT2440, Classwork12, Spring2025

ID: \_\_\_\_\_

Name: \_\_\_\_\_

## 1. Valid Arguments:

An argument is a sequence of propositions that end with a conclusion.

The premises are all but the final propositions in the argument and the final proposition is called a conclusion.

An argument is valid if the truth of all its premises implies that the conclusion is true.

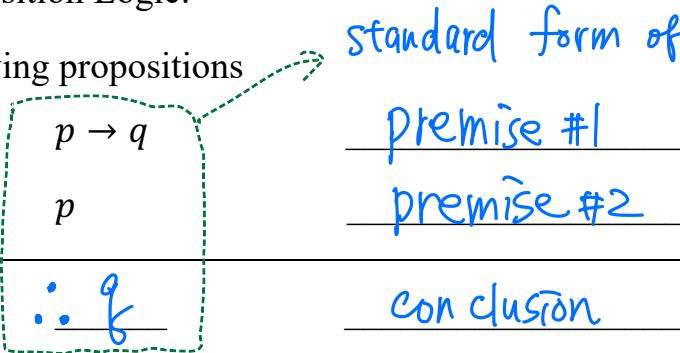
## 2. Example of a Valid Argument in Proposition Logic:

Consider the following argument involving propositions

“If it snows, I will go skiing.”

“It shows”

Therefore, I will go skiing.



$p$	$q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$p \wedge (p \rightarrow q) \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is a valid argument form. It means by knowing the two premises P and  $p \rightarrow q$ , we can conclude q, that is,  $(P \wedge (p \rightarrow q)) \rightarrow q$  is a tautology.

3. The Rules of Inference for Propositional Logic:

*deductive argument*

Name	Rule of Inference	Tautology
(1) Modus ponens <i>implication elimination</i>	$\begin{array}{c} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$	$p \wedge (p \rightarrow q) \rightarrow q$
(2) Modus tollens $p \rightarrow q \equiv \neg q \rightarrow \neg p$ <i>(contraposition)</i>	$\begin{array}{c} p \rightarrow q \\ \neg q \\ \hline \therefore \neg p \end{array}$	$(\neg q) \wedge (p \rightarrow q) \rightarrow (\neg p)$
(3) Hypothetical syllogism	$\begin{array}{c} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
(4) Disjunctive syllogism	$\begin{array}{c} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$	$(\neg p \wedge (p \vee q)) \rightarrow q$
(5) Addition <i>If p is "true", then "p or Q" is true</i>	$\begin{array}{c} p \\ \hline \therefore p \vee q \end{array}$	$p \rightarrow p \vee q$ $q \rightarrow q \vee p$
(6) Simplification	$\begin{array}{c} p \wedge q \\ \hline \therefore p \quad \therefore q \end{array}$	$p \wedge q \rightarrow p$ $p \wedge q \rightarrow q$
(7) Conjunction	$\begin{array}{c} p \\ q \\ \hline \therefore p \wedge q \end{array}$	$((p) \wedge (q)) \rightarrow p \wedge q$
(8) Resolution	$\begin{array}{c} p \vee q \\ \neg p \vee r \\ \hline \therefore q \vee r \end{array}$	$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$