MAT2440, Classwork12, Spring2025

ID:______________________ Name:______________________

1. Valid Arguments:

An __argument__ is a sequence of propositions that end with a conclusion.

The __premises__ are all but the final propositions in the argument and the final proposition is called __a conclusion__.

An argument is __valid__ if the truth of all its premises implies that the conclusion is true.

2. Example of a Valid Argument in Proposition Logic:

Consider the following argument involving propositions

standard form of an argument.

''If it snows, I will go skiing.''	$p \to q$	premise #1
''It shows''	p	premise #2
Therefore, I will go skiing.	$\therefore q$	conclusion

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \to q}$	$\boldsymbol{p \wedge (p \to q)}$	$\boldsymbol{p \wedge (p \to q) \to q}$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

This is a __valid__ __argument__ form. It means by knowing the two premises __p__ and __$p \to q$__, we can conclude __q__, that is, __$(p \wedge (p \to q)) \to q$__ is a tautology.

3. The Rules of Inference for Propositional Logic: deductive argument

Name	Rule of Inference	Tautology
(1) Modus ponens implication elimination	$p \rightarrow q$ p ——— $\therefore q$	$p \wedge (p \rightarrow q) \rightarrow q$
(2) Modus tollens $p \rightarrow q \equiv \neg q \rightarrow \neg p$ (contraposition)	$p \rightarrow q$ $\neg q$ ——— $\therefore \neg p$	$(\neg q) \wedge (p \rightarrow q) \rightarrow (\neg p)$
(3) Hypothetical syllogism	$p \rightarrow q$ $q \rightarrow r$ ——— $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
(4) Disjunctive syllogism	$p \vee q$ $\neg p$ ——— $\therefore q$	$(\neg p \wedge (p \vee q)) \rightarrow q$
(5) Addition If p is "true", then "P or Q" is true	p ——— $\therefore p \vee q$ q ——— $\therefore q \vee p$	$p \rightarrow p \vee q$ $q \rightarrow q \vee p$
(6) Simplification	$p \wedge q$ ——— $\therefore p$ $p \wedge q$ ——— $\therefore q$	$p \wedge q \rightarrow p$ $p \wedge q \rightarrow q$
(7) Conjunction	p q ——— $\therefore p \wedge q$	$((p) \wedge (q)) \rightarrow p \wedge q$
(8) Resolution	$p \vee q$ $\neg p \vee r$ ——— $\therefore q \vee r$	$(p \vee q) \wedge (\neg p \vee r) \rightarrow q \vee r$