MAT2440, Classwork11, Spring2025

ID:_____ Name:_____

1. Translating English into Logic Expressions:

Let $I(x)$ be the statement ‘‘x has an Internet connection’’ and $C(x, y)$ be the statement ‘‘x and y have chatted over the internet,’’ where the domain for x and y consists of all students in your class. Use quantifiers to express each of these statements:

(a) Jerry doesn't have an internet connection. $\neg I(\text{Jerry})$

(b) Rachel has not chatted over the internet with Chelsea. $\neg C(\text{Rachel}, \text{Chelsea})$

(c) No one in the class has chatted with Bob. $\forall x \neg C(x, \text{Bob})$

$(\neg \exists x C(x, \text{Bob}))$

(d) Sanjay has chatted with everyone. $\forall y\, C(\text{Sanjay}, y)$

(e) Someone in your class doesn't have an internet connection. $\exists x \neg I(x)$

(f) Not everyone in your class has an internet connection. $\neg \forall x\, I(x)$

(g) There is a student in your class who has chatted with everyone in your class over the internet. $\exists x \forall y\, C(x, y)$

2. Some examples of the **Negating Nested Quantifiers**:

Express the negation of the statement $\forall x \exists y (xy = 1)$ so that no negation precedes a quantifier.

$$\neg (\forall x \exists y (xy = 1))$$
$$\equiv \exists x \neg (\exists y (xy = 1))$$
$$\equiv \exists x \forall y \neg (xy = 1)$$
$$\equiv \exists x \forall y (xy \neq 1)$$

3. Rewrite the statement so that negation only appearing within predicates

$$\neg(\exists x \forall y (P(x, y) \wedge Q(x, y)))$$
$$\equiv \forall x \neg \forall y (P(x,y) \wedge Q(x,y))$$
$$\equiv \forall x \exists y \neg (P(x,y) \wedge Q(x,y))$$
$$\equiv \forall x \exists y (\neg P(x,y) \vee \neg Q(x,y))$$