MAT2440, Classwork10, Spring2025

ID:________________________ Name:________________________

1. The De Morgan's laws for quantifiers: $x: x_1, x_2, x_3, x_4, \cdots, x_n$

When the domain of a predicate $P(x)$ consists of n elements, we have

$\neg\forall x P(x) \equiv \neg\left(P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge P(x_4) \wedge \cdots \wedge P(x_n)\right)$

De Morgan's $\Rightarrow$ $\equiv \neg P(x_1) \vee \neg P(x_2) \vee \neg P(x_3) \vee \neg P(x_4) \vee \cdots \vee \neg P(x_n)$

$\equiv \exists x\, \neg P(x)$, $x = x_1, x_2, x_3, \cdots, x_n$

$\neg\exists x P(x) \equiv \neg\left(P(x_1) \vee P(x_2) \vee P(x_3) \vee P(x_4) \vee \cdots \vee P(x_n)\right)$

De Morgan's $\Rightarrow$ $\equiv \neg P(x_1) \wedge \neg P(x_2) \wedge \neg P(x_3) \wedge \neg P(x_4) \wedge \cdots \wedge \neg P(x_n)$

$\equiv \forall x\, \neg P(x)$, $x = x_1, x_2, x_3, x_4, \cdots, x_n$

2. What are the negations of the given statements: domain of x is all real number

(a) $\forall x(x^2 > x)$ $\Rightarrow \neg \forall x(x^2 > x) \equiv \exists x\, \neg(x^2 > x) \equiv \exists x\,(x^2 \leq x)$

(b) $\exists x(x^2 = 2)$ $\Rightarrow \neg \exists x(x^2 = 2) \equiv \forall x\, \neg(x^2 = 2) \equiv \forall x\,(x^2 \neq 2)$

(c) $\forall x(1 < x < 3)$ $\Rightarrow \neg \forall x(1 < x < 3) \equiv \exists x\, \neg(1 < x < 3)$

$\equiv \exists x\,(x \geq 3 \text{ or } x \leq 1)$

3. Some examples of the **nested quantifiers**:

→ $P(x,y)$. predicates or propostional function

(a) $\forall x \forall y(x + y = y + x)$

It means " For every real number x and for every real number y , x+y equals y+x "

(This is the "commutative laws for addition")

(b) $\forall x \exists y(x + y = 0)$

It means " For every x , there exists a y such that x+y = 0" True

(c) $\exists y \forall x(x + y = 0)$

It means " There is a real number y for all x such that x+y = 0" False.

(d) $\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$

It means " For every x and for every y, if x > 0 and y > 0, then xy < 0"

" For every positive x and for every negative y, xy < 0"

4. Let $P(x, y)$ be the statement ''student x has taken class y,'' where the domain for $\boldsymbol{x}$ consists of **all the students in your class** and for $\boldsymbol{y}$ consists of **all the computer science courses at your school**. Express each of those quantifications:

(a) $\exists x \exists y P(x, y)$

It means " There is at least one student in your class who has taken at least one CS course in your school"

(b) $\exists x \forall y P(x, y)$

It means " There is at least one student in your class who has taken all CS course in your school

(c) $\forall x \exists y P(x, y)$

It means " Every student in your class has taken at least one CS course in your school"

(d) $\exists y \forall x P(x, y)$

It means " There is a CS course that every student in your class has taken"

(e) $\forall y \exists x P(x, y)$

It means " Every CS course has taken by at least one student in your class"

(f) $\forall x \forall y P(x, y)$

It means " Every student in your class has taken every CS course"