MAT2440, Quiz5, Spring2025

ID:_____ Name:_____

1. True or False.

T (a) Let $S = \{1, \{1\}\}$ be a set. Then $|S| = 2$.

T (b) $\overline{A \cup B} = \bar{A} \cap \bar{B}$. (De Morgan's law)

F (c) There exist sets A and B such that $A - B$ and $B - A$ are Not disjoint.

T (d) A function f is invertible if f is bijection.

F (e) Let $f: \mathbb{Z} \to \mathbb{Z}$ be such that $f(x) = 2x$. Then f is invertible.

↑ NOT AN ONTO function.

2. Let $f: \mathbb{R} \to \mathbb{R}$ be such that $f(x) = 3x + 5$. Is f invertible, and if it is, what is its inverse? If it is not invertible, why?

I. First, to show f is invertible, it is equivalent to show f is bijection:

① To show f is one-to-one. Cancel "5" Divided by "3"

Let $f(a)=f(b)$, we have $3a+5=3b+5 \Rightarrow 3a=3b \Rightarrow a=b$

Therefore, since the same output gets the same input, f is one-to-one

② To show f is onto. Let $y=f(x)$, we have $y=3x+5$

$\forall y \in \mathbb{R}$, we have $y=3x+5 \Rightarrow 3x=y-5 \Rightarrow x=\frac{y-5}{3} \in \mathbb{R}$

(since for all real numbers y, $\frac{y-5}{3}$ is real and it implies x is a real number for all y)

Therefore, all output can find an input which implies f is onto.

By ① & ②, f is bijection. Thus, f is invertible.

II. To find f^{-1}, we have

① Let $y=f(x)$: $y=3x+5$

② Switch x & y : $x=3y+5$

③ Solve for y : $3y=x-5 \Rightarrow y=\frac{x-5}{3}$

④ replace y by $f^{-1}(x)$

$\Rightarrow f^{-1}(x)=\frac{x-5}{3}$.